TWO-DIMENSIONAL COLLISIONS

OBJECTIVE

Investigate elastic and inelastic collisions between two objects on a plane.

SUMMARY

In any collision between two bodies, the colliding objects must obey the laws of conservation of energy and conservation of momentum. With the help of these two conserved quantities it is possible to describe how the bodies will behave after the collision. In the case of a flat plane, the velocity and momentum need to be expressed as vectors. A particularly simple description can be obtained by switching to a system which focuses on the mutual centre of gravity of the two bodies. In this experiment, two discs of specific mass are allowed to collide on an air cushion table and the velocities are then recorded with the aid of a spark generator.

EXPERIMENT PROCEDURE

- **Determine the velocities before and after a collision.**
- **Verify the conservation of momentum for elastic and inelastic collisions.**
- **Verify the conservation of energy for elastic and inelastic collisions.**
- **Investigate the motion of the centres of gravity in the system.**

REQUIRED APPARATUS

Quantity	Description	Number
1	Air Cushion Table (230 V, 50/60 Hz)	**1013210 or**
	Air Cushion Table (115 V, 50/60 Hz)	**1012569**
1	Pair of Magnetic Pucks	**1003364**
Additionally recommended		
1	Mechanical Balance 610	**1003419**
1	Ruler, 50 cm	
1	Goniometer	

BASIC PRINCIPLES

A collision refers to a brief interaction between two bodies. It is assumed that this interaction takes place in the space of a certain, short length of time and that the bodies do not affect one another in any other way. If no other forces are present, the two bodies will move at constant velocities both before and after the collision. Since the two bodies may be regarded as a closed system, the interaction must obey the laws of conservation of momentum and conservation of energy.

The velocities of bodies 1 and 2 before the collision are represented by the vectors v_1 and v_2. Those after the collision are represented by v'_1 and v'_2. The corresponding momentum is represented by p_i and p'_i (i = 1, 2). The masses of both bodies remain constant over time and are labelled m_1 and m_2. Due to conservation of momentum, the following must be true:

(1) $$m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v'_1 + m_2 \cdot v'_2$$

In addition, when the collisions are elastic, the overall kinetic energy in the system is also conserved:

(2) $$\frac{1}{2} \cdot m_1 \cdot v_1^2 + \frac{1}{2} \cdot m_2 \cdot v_2^2 = \frac{1}{2} \cdot m_1 \cdot v'^2_1 + \frac{1}{2} \cdot m_2 \cdot v'^2_2$$

If body 2 is at rest before the collision, it is possible to select a coordinate system in which the motion of body 1 is along the x-axis ($v_{1y} = 0$). This does not in any way affect the generality of the description.

First let us consider a collision in line with the centres of gravity of both objects, where $d = 0$, see Fig. 1. The bodies will then move along the x-axis and the velocities after the collision are given by:

(3) $$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot v_1$$

and

(4) $$v'_2 = \frac{2m_1}{m_1 + m_2} \cdot v_1$$

For identical masses, $m_1 = m_2$, the following conditions are true:

(5) $$v'_1 = 0$$

and

(6) $$v'_2 = v_1$$

If collisions are off-centre but the masses are the same, the bodies will separate from one another at an angle of 90°, i.e.

(7) $$\theta_1 + \theta_2 = 90°$$

Additionally, if $v_{1y} = 0$ and $m_1 = m_2$, then equation (1) provides the following result:

(8) $$v'_{1y} = -v'_{2y}$$

The position vector for the centre of gravity is as follows:

(9) $$r_s = \frac{m_1 \cdot r_1 + m_2 \cdot r_2}{m_1 + m_2}$$

Since the total momentum is conserved, the velocity of the centre of gravity is constant and is given by the following equation:

(10) $$v_s = \frac{m_1 \cdot v_1 + m_2 \cdot v_2}{m_1 + m_2}$$

The total momentum corresponds to the momentum of a single mass $m_S = m_1 + m_2$, which moves at the same velocity as the centre of gravity.

It often makes sense to transform the frame of reference to a system centred on the combined centre of gravity of the two bodies. Then, before the collision, the two bodies will converge towards one another in such a way that the overall momentum is zero. After an elastic collision, they then separate in such a way that the total momentum continues to be zero. After a completely inelastic collision, they stick together and rotate about their mutual centre of gravity. The kinetic energy of the system is also conserved in this case. In this experiment, two discs of known mass are allowed to collide on a cushion of air. The motion they undergo is recorded with the help of a spark generator.

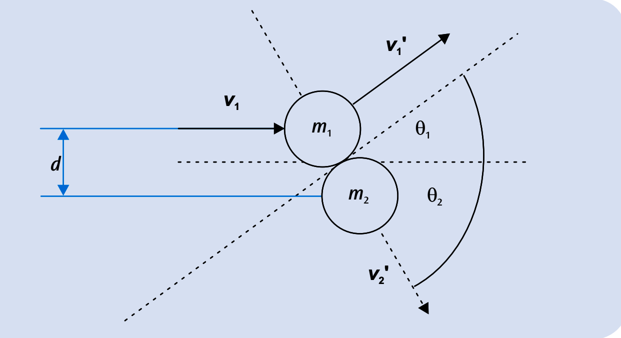

Fig. 1: Schematic representation of an off-centre collision between two bodies

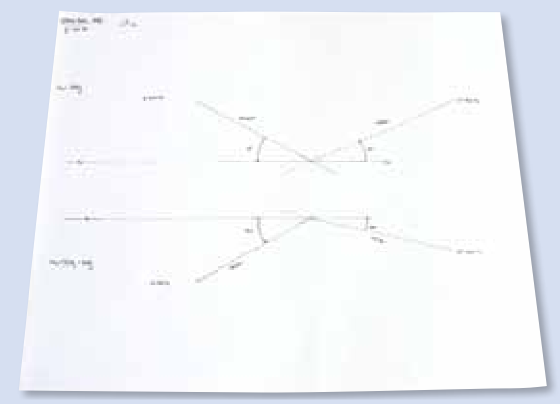
Fig. 2: Recording and evaluation of an off-centre collision between two bodies of unequal mass and initial velocities $v_1 \neq 0$ and $v_2 \neq 0$

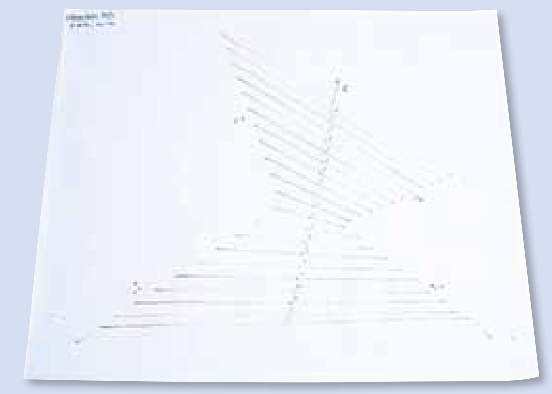
Fig. 3: Position of centre of gravity S

EVALUATION

Calculation of the kinetic energy indicates that some energy is lost. This is due to the sound made upon collision, the slight deformation of the bodies when they collide, any intrinsic rotation of the discs which has not been taken into account and movement of the hoses feeding the cushion of air.

The magnitude of the velocities can be calculated using the following relationship: $v = \Delta \cdot f$,

Δ Distance between two points,

f: Frequency of spark generator

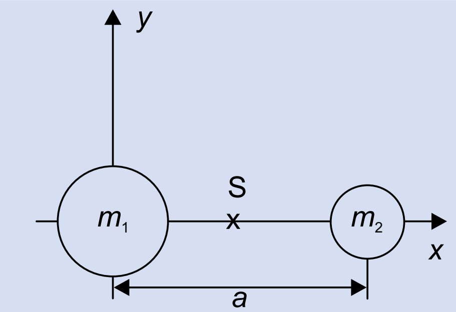

Fig. 4: Motion of centre of gravity S before and after collision