MECHANICS / OSCILLATIONS

UE1050250

FOUCAULT PENDULUM



- Measure the direction of the oscillation as a function of time.
- Determine the speed of the rotation.
- · Determine the latitude where the experiment is taking place.

OBJECTIVE Demonstrate the rotation of the earth with a Foucault pendulum

SUMMARY

A Foucault pendulum is a long string pendulum with a heavy bob, which can be used to demonstrate the rotation of the earth. In this experiment, a pendulum 1.2 metres in length is used. The direction of its oscillations can be very accurately determined by projecting the pendulum's shadow. For long periods of observation, any damping of the oscillation can be compensated for with the aid of an adjustable electromagnetic system to provide additional momentum.



REQUIRED APPARATUS

uantity	Description	Number	
1	Foucault Pendulum (230 V, 50/60 Hz)	1000748	or
	Foucault Pendulum (115 V, 50/60 Hz)	1000747	
1	Digital Stopwatch	1002811	

BASIC PRINCIPLES

(1)

(2)

(4b)

A Foucault pendulum is a long string pendulum with a heavy bob, which can be used to demonstrate the rotation of the earth. It is named after Jean Foucault, who in 1851 discovered that the direction of the oscillation of a pendulum 2 m in length would change over the course of time. This experiment was later repeated with ever longer and heavier pendulums.

Since the earth rotates on its axis, when using an earth-based coordinate system, a force called the Coriolis force arises, which then acts on the moving pendulum in a direction perpendicular to the direction of the oscillation:

> $\mathbf{F} = 2 \cdot \mathbf{m} \cdot \mathbf{\Omega}_0 \mathbf{x} \mathbf{v}$ m: mass of pendulum bob Ω_0 : vector describing angular velocity of earth v: velocity vector of oscillating pendulum

This causes the plane of the oscillation to turn with an angular frequency dependent on the angle of latitude φ of the point from which the pendulum is suspended:

Because the Foucault pendulum is only deflected by a small angle α , the pendulum bob can be assumed to move exclusively in the horizontal plane, which can be seen in Fig. 1, and moves between an axis N aligned with north and an axis E aligned with east. The observation is concerned only with horizontal deflections since the ppendulum bob is hanging from a thread. For this reason, only the vertical component of the veactor Ω_0 is relevant:

 $\Omega(\phi) = \Omega_0 \cdot \sin\phi$

The equation of motion for an oscillating Foucault pendulum is therefore as follows:

(3)
$$\frac{d^2\alpha}{dt^2} \cdot \boldsymbol{e}_p + 2 \cdot \Omega_0 \cdot \sin \varphi \cdot \frac{d\alpha}{dt} \cdot \boldsymbol{e}_v + \frac{g}{L} \cdot \alpha \cdot \boldsymbol{e}_p = 0$$

L: length of pendulum, *g*: acceleration due to gravity e_n: horizontal unit vector parallel to the current direction of oscillation \boldsymbol{e}_{v} : horizontal unit vector perpendicular to current direction of oscillation

The solution to this can be separated into a solution for the angle of deflection α and a solution for the turning unit vector \boldsymbol{e}_n parallel to the current direction of oscillation:

(4a)
$$\alpha(t) = \cos(\omega \cdot t + \beta)$$
 where $\omega = \sqrt{\frac{g}{L}}$

 $\boldsymbol{e}_{\mathrm{p}}(t) = \boldsymbol{e}_{\mathrm{E}} \cdot \cos(\psi(t)) + \boldsymbol{e}_{\mathrm{N}} \cdot \sin(\psi(t))$ where $\psi(t) = \Omega_0 \cdot \sin \phi \cdot t + \psi_0$: direction of oscillation $e_{\rm F}$: horizontal unit vector aligned with east e_{N} : horizontal unit vector aligned with north

The plane of the oscillation therefore rotates over the course of time with a frequency as given by equation (2). In the northern hemisphere the rotation is clockwise and in the southern hemisphere it is anti-clockwise. The speed of the rotation is at its highest at the poles, whereas at the equator there is no rotation at all.

In this experiment, a pendulum 1.2 metres in length is used. In order to avoid the oscillations becoming elliptical, the pendulum thread is allowed



to collide with a so-called Charon ring every time it swings. The direction of oscillation can be seen by projecting the shadow of the thread onto an angle scale whereby the angle can be read off with great accuracy. It is possible to observe the rotation of the plane of oscillation after only a few minutes. For long periods of observation, any damping of the oscillation can be compensated for with the aid of an adjustable electromagnetic system to provide additional momentum.

EVALUATION

The angle of the oscillation plane ψ is in linear proportion to the time, see Fig. 2. The value we are seeking $\Omega(\phi)$ is the gradient of the straight lines through the measurements.

The latitude in degrees can be calculated by rearranging equation (2):

$$\varphi = \frac{180^{\circ}}{\pi} \cdot \arcsin\left(\frac{86400 \text{ s}}{360 \text{ grd}} \cdot \Omega(\varphi)\right)$$



Fig. 1: Illustration of Foucault pendulum in fixed earth-based coordinate system



Fig. 2: Measured curve recorded at latitude $\varphi = 50^{\circ}$