Variable-g pendulum

MEASURE THE PERIOD OF AN OSCILLATING PENDULUM AS A FUNCTION OF THE EFFECTIVE ACCELERATION DUE TO GRAVITY.

- Measure the period T as a function of the effective acceleration g_{eff}
- Measure the period T for various pendulum lengths L.

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BASIC PRINCIPLES

The period of oscillation of a mathematical The period of a pendulum is determined mathematically by the length of the pendulum L and the acceleration due to gravity g. The effect of the gravitational acceleration can be demonstrated by tilting the axis of the pendulum so that it is no longer horizontal.

When the axis is tilted, the component of the gravitational acceleration g that is parallel to the axis g_{par} is rendered ineffective by the fact that axis is fixed (see Fig.). The remaining component that is effective g_{eff} is given by the following equation:

$$g_{eff} = g \cdot \cos\alpha \tag{1}$$

where α is the angle of inclination of the axis to the horizontal.

When the pendulum is deflected by an angle φ from its rest position, a suspended weight of mass m experiences a returning force of the following magnitude:

$$F = -m \cdot g_{eff} \cdot \sin\varphi \tag{2}$$

For small angles the equation of motion of the pendulum emerges as the following:

$$m \cdot L \cdot \ddot{\varphi} + m \cdot g_{eff} \cdot \sin\varphi = 0 \tag{3}$$

The pendulum's angular frequency of oscillation is therefore:

$$\omega = \sqrt{\frac{g_{eff}}{L}} \tag{4}$$

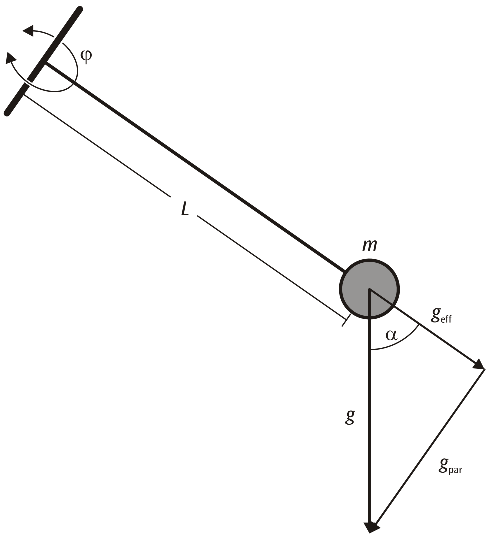

Fig. 1: Variable-g pendulum (photograph and schematic diagram)

LIST OF APPARATUS

1	Variable-g pendulum	U11984
1	Mechanical stopwatch (30 seconds)	U40801
1	Stand base, 150mm	U13270
1	Stand rod, 470mm	U15002

SET-UP

- Mount the variable-g pendulum in the stand base.
- Attach the mass to the lower end of the pendulum rod.

EXPERIMENT PROCEDURE

- Keeping the length of the pendulum L at its maximum, set varying angles of inclination α and measure the time for 10 oscillations in each case.
- At an angle of inclination $\alpha = 70°$, vary the length of the pendulum by shifting the mass and measure the time for 10 oscillations in each case.

SAMPLE MEASUREMENTS

a) Variation of the angle of inclination:

Table 1: Period of oscillation of the pendulum in relation to the angle of inclination α of the axis of oscillation, or in relation to the effective component of gravitational acceleration $g \cos \alpha$ (L = 30cm) which can be calculated from equation (1).

α	$g \cos \alpha$ (ms^{-2})	$10T$ (s)	T (s)
0°	9.81	11.0	1.10
10°	9.66	11.1	1.11
20°	9.22	11.4	1.14
30°	8.50	11.8	1.18
40°	7.51	12.6	1.26
50°	6.31	13.9	1.39
60°	4.91	15.9	1.59
70°	3.36	19.5	1.95
80°	1.70	27.2	2.72
85°	0.85	36.8	3.68

b) Variation of the length of the pendulum at $\alpha = 70°$

Table 2: Period of oscillation of the pendulum in relation to the length of the pendulum L ($g \cos \alpha$ = 3.36ms^{-2})

L (cm)	$10T$ (s)	T (s)
30	19.5	1.95
20	16.2	1.62
10	12.8	1.20

EVALUATION

Using equation (4), we can calculate the period of oscillation of the pendulum:

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

For L = 30cm, we get the continuous curve depicted in Fig. 2. The points plotted in Fig. 2 are also taken from Table 1 and coincide with the curve in terms of measurement accuracy.

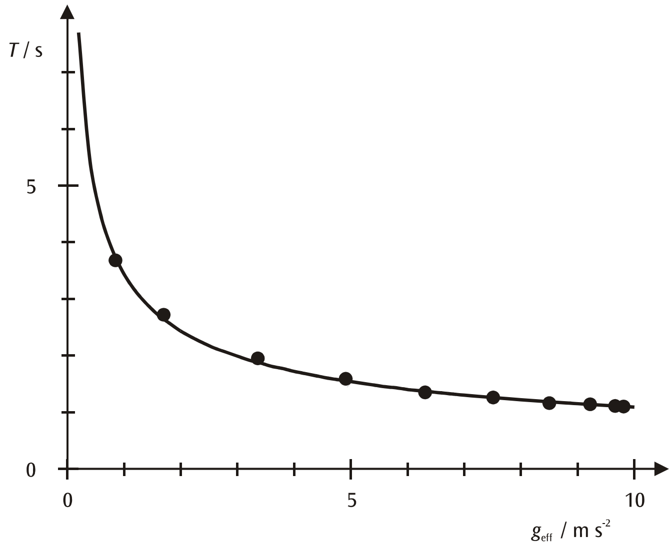

Fig. 2: Period of oscillation of the pendulum in relation to the effective acceleration due to gravity

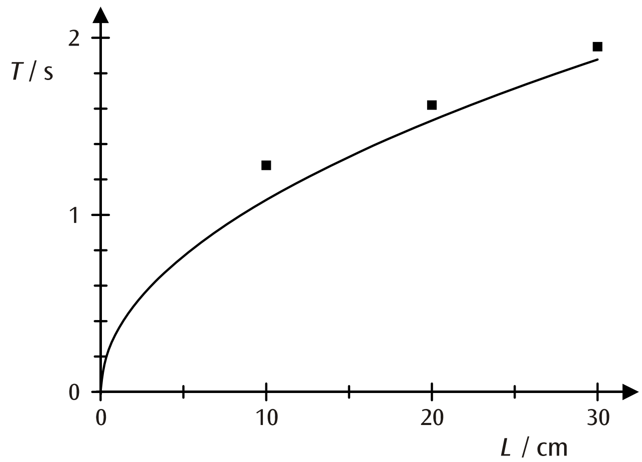

Fig. 3: Period of oscillation of the pendulum in relation to its length L

The continuous curve in Fig. 3 has been calculated by taking the value of the effective component g_{eff} = 3.36ms^{-2}. The measured values are taken from Table 3 and deviate from the plotted curve, since the pendulum deviates perceptibly from the mathematical ideal for short lengths L.

RESULTS

The period of oscillation of the pendulum becomes shorter when the length of the pendulum is reduced and it becomes greater when the effective component of gravitational acceleration is reduced.