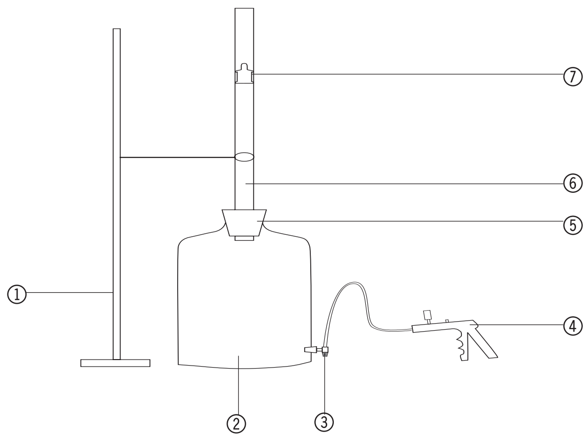
U14328 Precision tube

Instruction sheet

9/03 ALF



① *Stand*
② *Marriott bottle*
③ *3-way stopcock*
④ *Hand pump*
⑤ *Stoppers*
⑥ *Precision tube*
⑦ *Aluminium cylinder*

The precision tube is used together with the Marriott bottle U14327 to determine the adiabatic exponent c_p/c_V using Rüchardt's method.

1. Safety instructions

- Handle the glass tube carefully. Do not subject to mechanical stress or it may break.
- Make sure both glass tube and aluminium cylinder are thoroughly clean because even the slightest amount of dirt leads to increased friction.
- Do not drop the aluminium cylinder. Even the slightest deformation adversely affects the experiment.

2. Description, technical data

A precision glass tube supplied with rubber stoppers at both ends and with an aluminium cylinder precisely engineered to slip inside.
If the glass tube is held vertical with the bottom end closed and the aluminium cylinder is allowed to slide down inside it, the cylinder drops very slowly since there is only a tiny gap around the sides for air to escape from underneath. If the tube is rotated by 180°, the cylinder behaves in the same manner since the pressure in the upper part of the tube is reduced as the cylinder slides down, and air can only get in to fill the space very slowly. In the third situation, the cylinder is allowed to drop into an open tube which is then immediately stoppered. The cylinder then slows and oscillates up and down a few times.

Dimensions:	600 mm x 16 mm $Ø_{inner}$
Aluminium cylinder:	15.2 g

2.1 Scope of supply
1 Precision glass tube
2 Rubber stoppers
1 Aluminium cylinder

3. Theory

Symbols used in the formula:

m: Mass of aluminium cylinder
d: Inside diameter of precision tube
A: Cross-sectional area of precision tube
V: Volume of measuring flask
p_L: Atmospheric pressure
p: Pressure in the bottle
g: Acceleration due to Earth's gravity
n: Number of moles
R: Universal gas constant (8.31451 kJ/kmol K)
T: Temperature
T_s: Period of oscillation
t: Time
c_p: Specific heat at constant pressure
c_V: Specific heat at constant volume
χ: c_P/c_V
ω: Natural frequency of oscillation

The state of an enclosed quantity of an ideal gas can be uniquely expressed in terms of the quantities pressure p, volume V and temperature T as follows:

$$p\,V = n\,R\,T \qquad (1)$$

For changes in state where no exchange of heat with the environment takes place, this equation can be reduced to the adiabatic formula:

$$p\,V^{\chi} = \text{const.} \qquad (2)$$

The adiabatic exponent χ is the ratio of the specific heat at constant pressure c_p to the specific heat at constant volume c_V:

$$\chi = \frac{c_P}{c_V} \qquad (3)$$

A stopper with a hole is put into a glass vessel with a volume of 10 *l*. The precision tube runs through the hole in this stopper so that it is in a vertical position. If the aluminium cylinder is allowed to drop into the tube, it bounces on the cushion of air enclosed inside the apparatus. This leads to a periodic oscillation.
When the pressure p in the glass vessel is equal to the sum of the pressure due to the mass of the aluminium cylinder and the external atmospheric pressure, the cylinder is in a state of equilibrium:

$$p = p_L + \frac{mg}{A} \qquad (4)$$

If the cylinder is moved a distance s from its equilibrium position, p changes by the value Δp and V changes by ΔV. A force acts on the aluminium cylinder to push it back towards equilibrium. This force is proportional to the distance s. A harmonic oscillation now begins atop the air cushion under the cylinder. Since the oscillation occurs fairly rapidly, it can be described in terms of the adiabatic change of state. By deriving dp/dV from equation (2) and assuming this also applies to the small finite changes Δp and ΔV we obtain

$$\Delta p = -\chi \frac{p}{V} \Delta V \qquad (5)$$

Since the cylinder moves along a distance s in the precision tube, the change in volume is

$$\Delta V = As \qquad (6)$$

The restoring force

$$F = A\Delta p = -\chi \frac{pA^2}{V} s \qquad (7)$$

leads to the periodic acceleration of a cylinder of mass m. Newton's second law then gives us the following differential equation for $s(t)$

$$\frac{d^2 s}{dt^2} + \chi \frac{pA^2}{V} s = 0 \qquad (8)$$

From (8) the natural frequency of oscillation is given by

$$\omega = \sqrt{\chi \frac{pA^2}{V}} \qquad (9)$$

thus the period of the oscillation T_s is

$$T_S = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{mV}{\chi p A^2}} \qquad (10)$$

Therefore, to derive the adiabatic exponent χ the following applies:

$$\chi = \frac{4\pi^2 mV}{A^2 p T_S{}^2} = \frac{64 mV}{T_S{}^2 d^4 p} \qquad (11)$$

4. Operation

- Determine the atmospheric pressure, the diameter of the inside of the precision tube, the mass of the aluminium cylinder and the volume of the measuring flask.
- Place the glass tube on the Marriott bottle, make sure it is vertical and secure it to a stand.
- The Marriott bottle should have a rubber mat or similar placed inside to avoid damage to the bottle and the cylinder if the cylinder falls into the bottle.
- To simplify the experiment, it is recommended that a hand pump be attached to the Marriott bottle via the 3-way stopcock. The cylinder can then be pumped back up the tube and retrieved from the top so that it is not necessary to repeatedly set the tube up.
- The aluminium cylinder should be cleaned with a fluff-free cloth and some petroleum ether. It should not be introduced into the tube at a crooked angle and allowed to drop when the stopcock is closed. Do not touch the cylinder except by the grip to prevent it getting dirty.
- Measure the duration of 5 oscillations 10 times using a stopwatch.
- The time measurement should start at the point when the cylinder comes to a halt for the first time at its lowest point. The watch should be stopped when the cylinder reaches its lowest point for the sixth time.
- Using the hand pump with the stopcock open, pump the cylinder back to the top. Make sure that the cylinder does not fall out at the end and get damaged.
- Take the cylinder right out of the tube so that the pressure in the apparatus returns to atmospheric pressure. Close the stopcock again.
- Make the measurements another nine times and determine the average value for the time.
- Perform the calculation.

General notes:

The accuracy of the measurements depends strongly upon the following factors:

- The precision tube must be extremely clean. If necessary, clean the tube with tissue paper.
- The aluminium cylinder must also be extremely clean. The slightest amount of dirt such as grease from fingers can lead to considerable friction. Therefore the cylinder should be cleaned before each measurement using a fluff-free cloth and some petroleum ether.
- The slightest deformation of the cylinder (e.g. due to having been dropped) adversely affects the experiment.
- The glass tube must be vertical.
- All stoppers must be airtight.
- Since the duration of the oscillation is squared in the equation, the time does need to be measured as accurately as possible.

5. Measurement example

Volume V:	10400 cm³
Mass of cylinder m:	15.2 g
$Ø_{inner}$ of tube d:	16 mm

Atmospheric pressure p_L: 1018 mbar

Time t in seconds for five oscillations:

	5.172
	5.276
	5.259
	5.224
	5.305
	5.175
	5.231
	5.241
	5.191
	5.175
Total:	52.249

Average: 5.2249

Period of oscillation T_s: 1.04498 s

Substituting into equation (11) gives:

$\chi = 1.39$

From published tables:

$\chi = 1.40$