SPINNING MOTION

PRECESSION AND NUTATION OF A GYROSCOPE

EXPERIMENT

PROCEDURE

• Verify that the frequency of rotation \( f_0 \)
  of a rotating disc is proportional to the
  period of precession of a gyroscope \( T_p \)
  and determine the moment of inertia by
  plotting \( f_0 - T_p \).

• Verify that the frequency of rotation \( f_n \)
  of a rotating disc is proportional to the
  frequency of nutation \( f_n \) by plotting \( f_n - T_n \)
  or the corresponding periods \( T_n - T_p \).

OBJECTIVE

Experimental investigation of precession and nutation of a gyroscope and determination of moment of inertia.

SUMMARY

A spinning disc exhibits motion known as precession and nutation in addition to its rotational motion, depending on whether there is an external force, and thereby an additional torque, acting upon its axis or if the axis of a disc spinning in an equilibrium state is then deflected from its equilibrium position. The period of precession is inversely proportional to the period of rotation while the period of nutation is directly proportional to the period of rotation. The way the period of precession depends on the period of rotation makes it possible to determine the moment of inertia of the rotating disc.

REQUIRED APPARATUS

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<th>Quantity</th>
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<td>Photo Gate</td>
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GENERAL PRINCIPLES

A spinning top is a rigid body which spins around an axis fixed at a given point. If an external force acts upon the axis, its torque causes a change in the angular momentum. The top then moves in a direction perpendicular to the axis and the force acting upon it. Such a motion is called precession. If a top is pushed away from its axis of rotation it starts to undergo a tipping motion. This motion is called nutation. In general, both these motions occur superimposed on one another.

In this experiment, a gyroscope is used rather than a top. Its large rotating disc rotates with low friction about an axis which is fixed at a certain bearing point. A counterweight is adjusted in such a way that the bearing point coincides with the centre of gravity. If the gyroscope is in equilibrium and the disc is set spinning, the moment \( I \) will be constant:

\[ L = I \omega \]

\( L \): moment of inertia, \( \omega \): angular velocity

The moment of inertia of the rotating disc of the gyroscope is given by:

\[ I = \frac{1}{2} M R^2 \]

\( M \): mass of disc, \( R \): radius of disc.

If extra weight is put on the axis of rotation by addition of a mass \( m \), the additional weight causes a torque \( \tau \) which changes the angular momentum:

\[ \tau = m \cdot g \cdot r \frac{dr}{dt} \]

where \( r \): distance from bearing point of axis of rotation to where the weight of the additional mass acts.

The axis of rotation then moves as shown in Fig. 2 by the following angle:

\[ df = \frac{m \cdot g \cdot r \frac{dr}{dt}}{L} \]

It also starts to precess. The angular velocity of the precession motion can then be derived:

\[ \omega_n = \frac{m \cdot g \cdot r \frac{dr}{dt}}{L} \]

where \( \omega_n \): angular velocity of the nutation.

If the disc is set spinning in the absence of any extra external torque and the axis of rotation is slightly deflected to one side, the gyroscope will exhibit nutation. The angular velocity of the nutation is then directly proportional to the angular velocity of the rotation:

\[ \omega_n = C \omega_p \quad \text{and} \quad T_n = C - T_p \]

C: constant

This experiment involves racing the rotational, preceptive and nutative motions with the help of photoelectric light barriers, whereby the way the pulses change over time is recorded and displayed by 3B NETlog™ and 3B NETlab™ units.

EVALUATION

The periods of rotation, precession and nutation are determined from the recordings of how the pulses change over time. According to equation (6), the period of precession is inversely proportional to that of the rotation, while \( T_n \) says that the period of rotation is directly proportional to that of the rotation. On the respective graphs, the measured values will therefore be along a straight line through the origin. From the slope of a line matched to these values \( T_n / T_p \) it is possible to obtain the moment of inertia of the gyroscope’s rotating disc by experiment and then compare it with the theoretical value calculated using equation (2).

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*Fig. 1: Schematic of a gyroscope illustrating precession
*Fig. 2: Schematic of a gyroscope illustrating nutation
*Fig. 3: Frequency of rotation \( f_n \) of a rotating disc as a function of the period of precession \( T_p \)
*Fig. 4: Period of rotation \( T_n \) as a function of period of rotation \( T_p \)