GRAVITATIONAL CONSTANT

EXPERIMENT PROCEDURE

- **Determine the initial equilibrium position of the torsional pendulum.**
- **Record the oscillation of the torsional pendulum about the final equilibrium position and determine the period.**
- **Determine where the final equilibrium position is.**
- **Calculate the gravitational constant *G*.**

OBJECTIVE

Measure the gravitational force and determine the gravitational constant using Cavendish torsion balance

SUMMARY

The central component of a Cavendish torsion balance is a sensitive torsional pendulum with a pair of small lead spheres attached to it. Two larger lead spheres are then placed near these two small balls in order to attract them. The position of the large spheres thus determines the equilibrium position of the torsional pendulum. If the two large spheres are then moved to a second position which is symmetrical with the first with respect to the two small balls, the torsional pendulum will adopt a new equilibrium position after a short period of settling. By measuring the geometry of the set-up in both positions, it is possible to determine the gravitational constant. The decisive factor in this is the equilibrium between the gravitational force and the restoring torque of the torsional pendulum. Measurements are made of the oscillation of the torsional pendulum using a capacitive differential sensor, which suppresses noise and vibrational components of the signal to a large extent. The tungsten wire from which the pendulum is made is chosen to be so thin that the period of oscillation is of the order of a few minutes, meaning that several oscillations about the equilibrium position may be observed in the space of an hour.

REQUIRED APPARATUS

Quantity	Description	Number
1	Cavendish Torsion Balance	**1003337**
1	Laser Diode, Red	**1003201**
1	Barrel Foot, 1000 g	**1002834**
1	Universal Clamp	**1002830**
1	Stainless Steel Rod 100 mm	**1002932**
Additionally recommended:		
1	Callipers, 150 mm	**1002601**
1	Electronic Scale 5000 g	**1003434**

BASIC PRINCIPLES

When measuring the gravitational force between two masses in a laboratory, it is inevitably the case that all other masses in the vicinity have a disturbing effect on the results. The Cavendish balance largely gets around this problem since two measurements are made with the masses symmetrically positioned.

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The gravitational force is given by the following:

(1) $$F = G \cdot \frac{m_1 \cdot m_2}{d^2}$$

G: Gravitational constant,
m_1: Mass of one small lead sphere,
m_2: Mass of one large lead sphere,
d: Distance between small and large lead spheres at the position where the measurement is made

The force deflects the torsional pendulum from its equilibrium position when the two large spheres are in position for the measurement. The deflecting torque is

(2) $$M_1 = 2 \cdot F \cdot r$$

r: Distance of small lead sphere from its mounting point on the supporting beam

If the torsional pendulum is deflected by an angle φ, there is a restoring torque

(3) $$M_2 = D \cdot \varphi$$

D: Torsion coefficient of tungsten wire

This acts due to the tungsten wire from which the support beam of the torsional balance is suspended. In the equilibrium position, M_1 and M_2 are equal.

The torsional coefficient D can be determined from the period of oscillation T for the oscillation of the torsional pendulum about its equilibrium position.

(4) $$D = J \cdot \frac{4\pi^2}{T^2}$$

The moment of inertia J comprises the moment of inertia J_1 of the two small spheres and the moment of inertia J_K of the supporting beam

(5) $$4 \cdot F \cdot r = D \cdot (\varphi - \varphi') = D \cdot \Delta\varphi$$

m_B: Mass of support beam
a, b: Length and width of support beam.

For the two large lead spheres, there should be two symmetrical positions where measurements are made. The angles of deflection in these two positions are φ and φ' and the two corresponding deflecting torques are equal but in opposite directions. In equilibrium, equations (2) and (3) therefore imply the following:

(6) $$J = 2 \cdot m_1 \cdot r^2 + \frac{m_B}{12} \cdot \left(a^2 + b^2\right)$$

In the course of the experiment the oscillations of the torsional pendulum are measured using a capacitive differential sensor, which suppresses noise and vibrational components of the signal to a large extent. The tungsten wire from which the pendulum is made is chosen to be so thin that the period of oscillation is of the order of a few minutes, meaning that several oscillations about the equilibrium position may be observed in the space of an hour. A mirror attached to the torsional pendulum can be used to set up a light pointer so that the oscillations are easy to follow with the naked eye. This makes the necessary adjustment and calibration of the balance much easier.

EVALUATION

By rearranging equations (1), (4), (5) and (6):

$$G = \frac{\Delta\varphi}{m_2} \cdot \frac{d^2 \cdot \pi^2}{T^2} \cdot \left(2 \cdot r + \frac{1}{12} \cdot \frac{m_B}{m_1} \cdot \frac{a^2 + b^2}{r}\right).$$

This does not take into account that the two small spheres are also attracted by the more distant large sphere, so that the torque on the torsional pendulum is somewhat reduced in comparison with the calculations made so far. It is not difficult to introduce correction for this into equation (2), since all the distance are known.

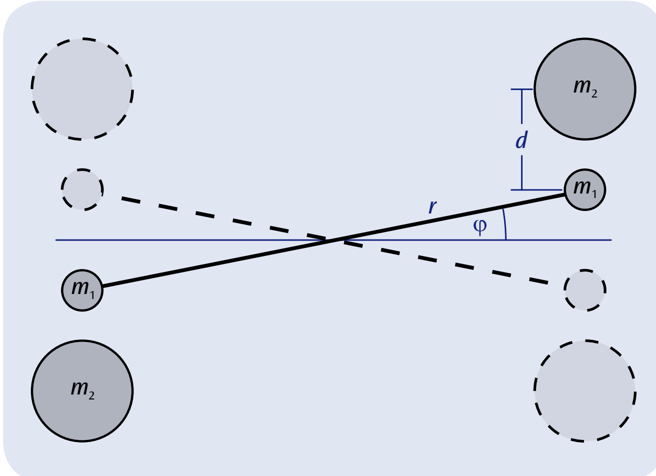

Fig. 1: Schematic of measurement set-up for the Cavendish torsional balance

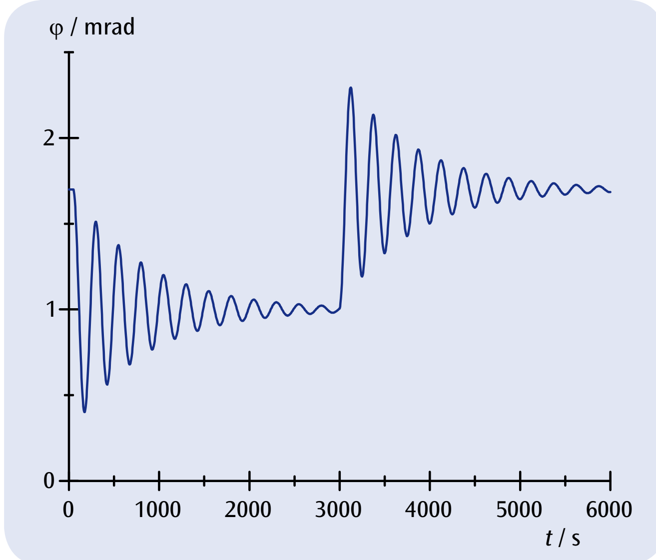

Fig. 2: Angle of deflection of torsional pendulum as a function of time when the measurement position of the two large lead spheres has been changed twice