PHYSICS EXPERIMENT

Kepler's Second Law

CONFIRM THE LAW OF EQUAL AREAS FOR CENTRAL FORCE MOTIONS (KEPLER'S SECOND LAW).

- Record the elliptical oscillations of a pendulum by the dust-marking method.
- Compare the velocities of the pendulum bob at the minimum and maximum distances from its rest position.
- Measure the areas swept by the radius vector of the pendulum in each time interval at the minimum and maximum distances from the rest position.

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BASIC PRINCIPLES

In the motion of a planet around the sun, the angular momentum remains constant, because the force acting on the planet is always directed towards the centre. From this it can be directly concluded that the planet's orbit must lie in a fixed plane. From it one can also derive Kepler's Second Law, the law of equal areas, which states that the light-beam from the sun to the planet sweeps equal areas in equal time intervals.

The validity of the law of equal areas is not affected by the exact form of the dependence of the central force on the distance from the force centre. This dependence only determines the shape of the orbit around the force centre. Thus, the equal areas law is also valid for the elliptical oscillations of a pendulum around the rest position, provided that the deflection angle from the vertical is not too great. The movement of the pendulum bob is almost exactly confined to a horizontal plane (see Fig. 1), and at any point on its path defined by the radius vector ***r*** there is a horizontal restraining force *F* directed towards the rest position, given by:

$$\boldsymbol{F} = -\frac{m \cdot g}{d} \cdot \boldsymbol{r}\,, \qquad (1)$$

g: gravitational acceleration,
d: length of pendulum, *m*: mass of pendulum bob.

The angular momentum is

$$L = m \cdot \boldsymbol{r}(t) \times \frac{\Delta \boldsymbol{r}(t)}{\Delta t} \qquad (2)$$

It thus remains unaffected by force ***F***. Therefore, the area ΔA swept by the radius vector ***r***(*t*) in each time interval Δt also remains constant:

$$\Delta A = \frac{1}{2} \cdot \left| \boldsymbol{r}(t) \times \Delta \boldsymbol{r}(t) \right| = \frac{1}{2} \cdot r(t) \cdot \Delta r(t) \cdot sin\alpha \qquad (3)$$

(see Fig. 2).

In the experiment the motion of the pendulum bob is recorded by the powder tracing method. In this method, the recording electrode attached to the pendulum bob is allowed to glide above an insulated tracing plate covered with fine sulphur powder. An alternating voltage at the mains frequency is applied between the electrode and the tracing plate, so that the sulphur powder is alternately attracted and repelled according to the changing polarity. This draws a trace consisting of time marks, and from the distances between these one can directly determine the velocity of the pendulum bob.

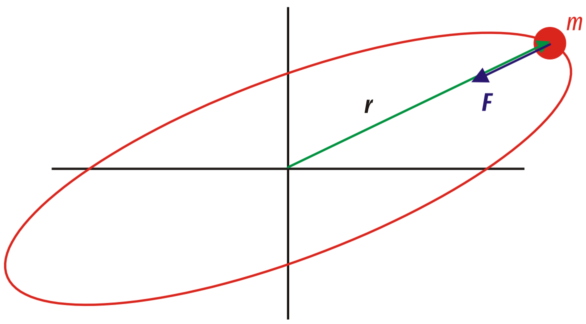

Fig. 1: Elliptical oscillation of the pendulum bob viewed from above

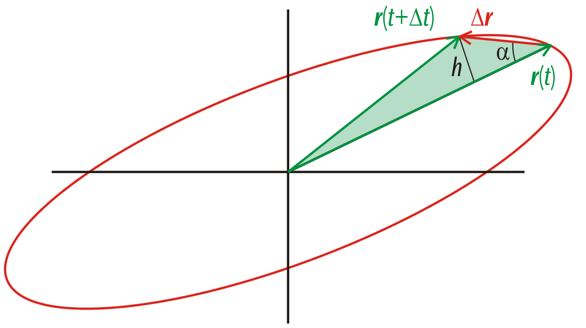

Fig. 2: The area swept by the radius vector of the pendulum bob in the time interval Δt

LIST OF APPARATUS

1 Equipment Set for Powder Tracing	1000739 (U8400870)
1 Pendulum with Plotting Electrode	1000780 (U8405640)
2 Tripod Stand, 150 mm	1002835 (U13270)
2 Stainless Steel Rod, 1000 mm	10002936 (U15004)
1 Stainless Steel Rod, 750 mm	1002935 (U15003)
3 Universal Clamp	1002830 (U13255)

SET-UP

Fig. 3: Experiment set-up

- Clamp both long stand rods vertically in the tripod bases and clamp the shorter rod horizontally near the top end of the two upright rods (see Fig. 3).
- With the brush, spread a thin layer of sulphur powder over the entire tracing plate as finely and uniformly as possible.
- Hang the pendulum by fixing the metal pin at the upper end of the pendulum chain in the third universal clamp.
- Lay the tracing plate between the two tripod bases and position the pendulum centrally above the tracing plate.
- Release the retaining catch of the tracing electrode.
- By shifting the horizontal rod vertically and turning the universal clamp around the rod, adjust the height of the pendulum so that the tracing electrode makes contact with the tracing plate even when the pendulum is deflected from its rest position.

EXPERIMENT PROCEDURE

- Connect the transformer to the mains supply.
- Insert one plug of the connecting lead into the socket of the powder tracing plate and clamp the other plug to a stand, ensuring that there is electrical contact.
- Give the pendulum bob a push to set it oscillating in an elliptical path with a large difference between the radii along the two main axes.
- Stop the pendulum bob and the tracing process when it has drawn one ellipse, or at most two.

SAMPLE MEASUREMENTS

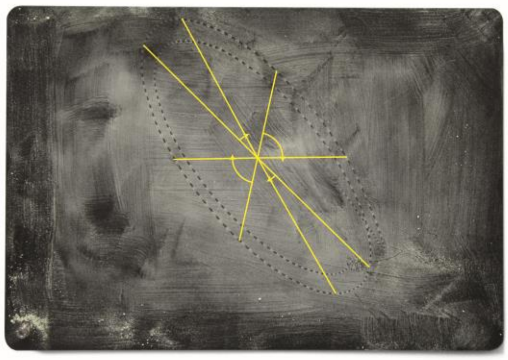

Fig. 4: Trace of the pendulum bob with mains-frequency time markers (the yellow lines show the areas swept by the radius vector during 10 time-marker intervals at minimum and maximum distance from the centre)

EVALUATION

First plot a graphical representation and determine the centre of the recorded path and the points on the orbit that correspond to the maximum and minimum distances from the centre. (see Fig. 4).

At each of these points on the orbit, it is possible to determine the area swept by the radius vector in 10 cycles of the alternating voltage (see Fig. 4). For purposes of simplicity, the areas are calculated by treating them approximately as triangles. This approximation neglects the contribution due to the curvature of the path. That contribution can be calculated in a second approximation, again as a triangle with the same base of length g (see Table 1).

The sum F of the areas of the two triangles from the first and second approximations gradually becomes smaller during the motion of the pendulum (see Table 1), as the angular momentum of the pendulum bob is gradually reduced by friction.

Table 1: Measurement of the area F swept by the radius vector in 10 time-marker intervals

	1st approximation		2nd approximation		Sum
g / mm	h_1 / mm	F_1 / mm²	h_2 / mm	F_2 / mm²	F / mm²
37	125	2310	7	130	2440
87	53	2310	3	130	2440
34	122	2070	7	120	2190
82	51	2090	2	80	2170