Mechanics

Spinning Motion

PHYSICS EXPERIMENT

Precession and Nutation of a Gyroscope

EXPERIMENTAL INVESTIGATION OF PRECESSION AND NUTATION OF A GYROSCOPE AND DETERMINATION OF MOMENT OF INERTIA

- **Verify that the frequency of rotation f_R of a rotating disc is proportional to the period of precession of a gyroscope T_P and determine the moment of inertia by plotting f_R (T_P).**
- **Verify that the frequency of rotation f_R of a rotating disc is proportional to the frequency of nutation f_N by plotting f_N (f_R) or the corresponding periods T_R (T_N).**

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Fig. 1: Set-up for measuring precession of a gyroscope (with no additional weight added).

GENERAL PRINCIPLES

A spinning top is a rigid body which spins around an axis fixed at a given point. If an external force acts upon the axis, its torque causes a change in the angular momentum. The top then moves in a direction perpendicular to the axis and the force acting upon it. Such a motion is called precession. If a top is pushed away from its axis of rotation its starts to undergo a tipping motion. This motion is called nutation. In general, both these motions occur superimposed on one another.

In this experiment, a gyroscope is used rather than a top. Its large rotating disc rotates with low friction about an axis which is fixed at a certain bearing point. A counterweight is adjusted in such a way that the bearing point coincides with the centre of gravity. If the gyroscope is in equilibrium and the disc is set spinning, the momentum *L* will be constant:

(1) $L = I \cdot \omega_R$

I: moment of inertia, ω_R: angular velocity

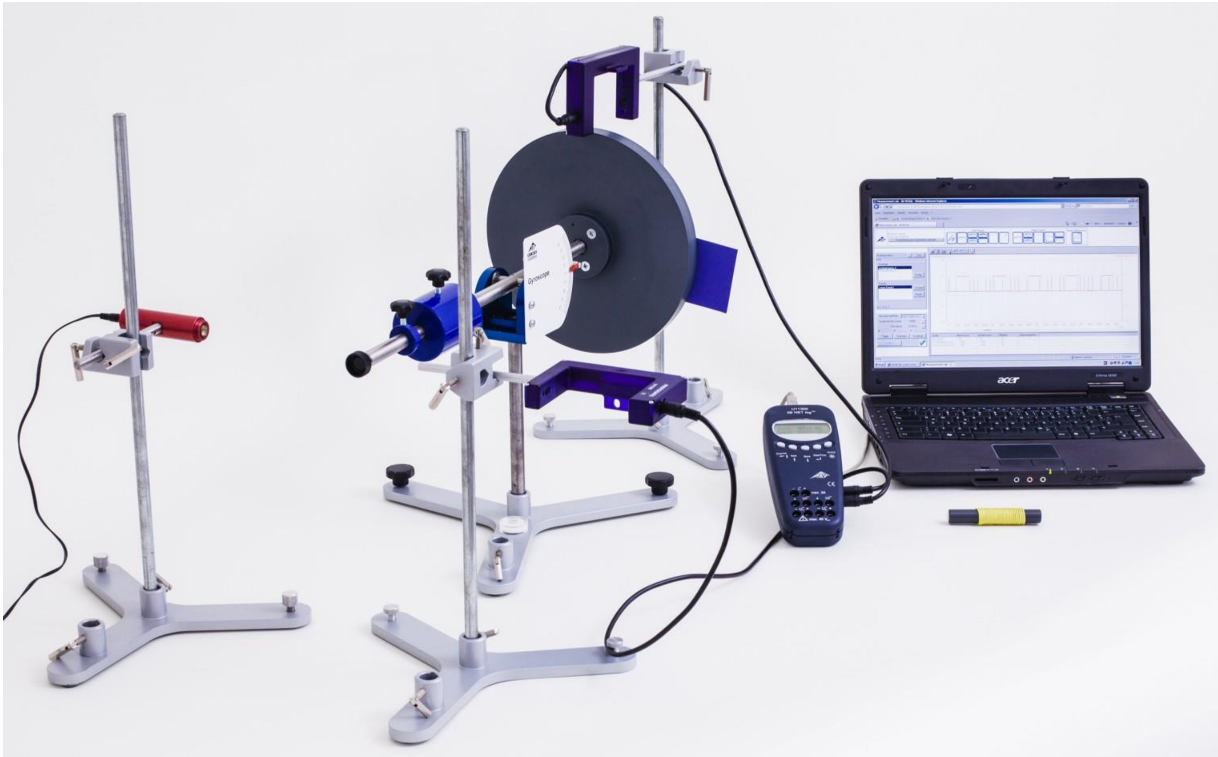

Fig. 2 Set-up for measuring nutation of a gyroscope.

The moment of inertia of the rotating disc of the gyroscope is given by:

(2) $I = \frac{1}{2} \cdot M \cdot R^2$

M: mass of disc, *R*: radius of disc

If extra weight is put on the axis of rotation by addition of a mass *m*, the additional weight causes a torque τ which changes the angular momentum:

(3) $\tau = m \cdot g \cdot r = \frac{dL}{dt}$

r: Distance from bearing point of axis of rotation to where the weight of the additional mass acts

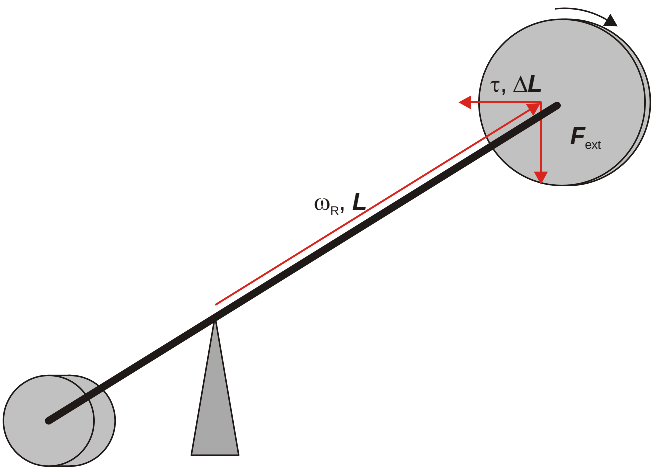

Fig. 3 Schematic of a gyroscope illustrating precession

The axis of rotation then moves as shown in Fig. 3 by the following angle:

(4) $d\phi = \frac{dL}{L} = \frac{m \cdot g \cdot r \cdot dt}{L}$

It also starts to precess. The angular velocity of the precession motion can then be derived:

(5) $\omega_P = \frac{d\phi}{dt} = \frac{m \cdot g \cdot r}{L} = \frac{m \cdot g \cdot r}{I \cdot \omega_R}$

where $\omega = 2\pi/T = 2\pi f$:

(6) $\frac{1}{T_R} = f_R = \frac{m \cdot g \cdot r}{I} \cdot T_P$

If the disc is set spinning in the absence of any extra external torque and the axis of rotation is slightly deflected to one side, the gyroscope will exhibit nutation. The angular velocity of the nutation is then directly proportional to the angular velocity of the rotation:

(7) $\omega_N = C \cdot \omega_R$ and $T_R = C \cdot T_N$

C: constant

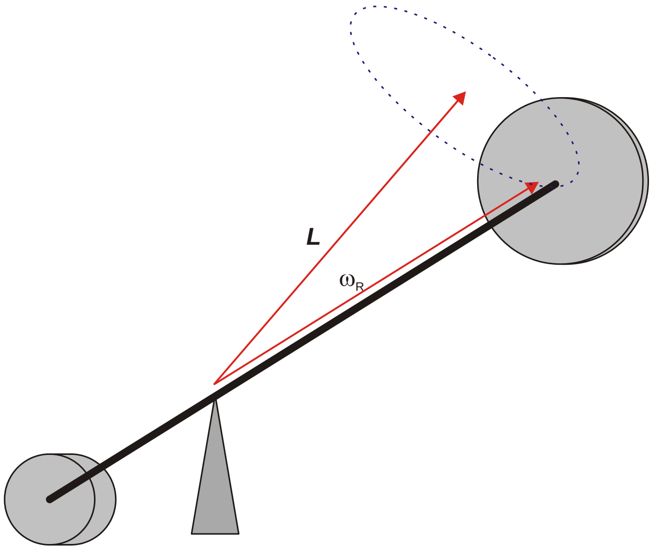

Fig. 4 Schematic of a gyroscope illustrating nutation

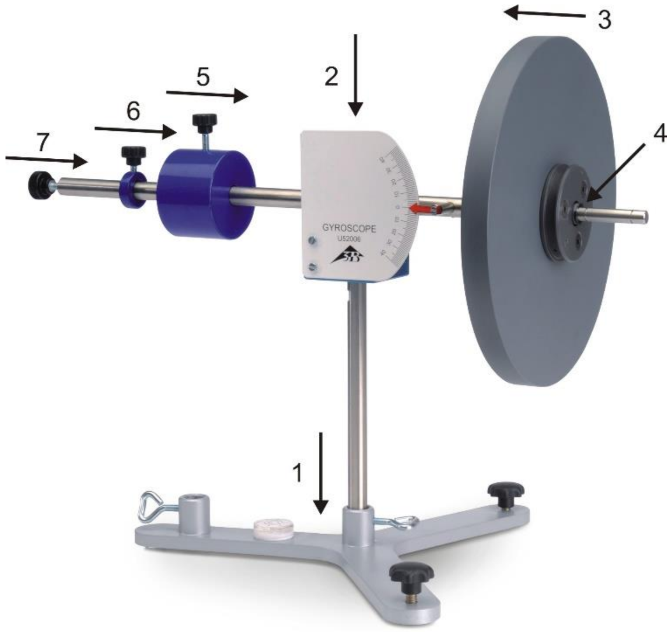

Fig. 5 Setting up the gyroscope.

LIST OF APPARATUS

1	Gyroscope	1000695 (U52006)
2	Photo Gate	1000563 (U11365)
1	Laser Diode, Red	1003201 (U22000)
1	3B NET*log*™ @230V	1000540 (U11300-230)
or		
1	3B NET*log*™ @115V	1000539 (U11300-115)
1	3B NET*lab*™	1000544 (U11310)
3	Tripod Stand 150 mm	1002835 (U13270)
3	Universal Clamp	1002830 (U13255)
3	Stainless Steel Rod 750 mm	1002935 (U15003)

SETTING UP A GYROSCOPE

- Place the stand base on a level working surface which is not subject to vibration and use the spirit level to align it to the horizontal.
- Insert the stand rod into the stand base and secure it (Fig. 5, 1).
- Place the rotor head in such a way that its main axle is on the stand rod (Fig. 5, 2).
- Move the flywheel onto the main gyroscope axle as far it will go with the bobbin end facing away from the rotor head (Fig. 5, 3). Insert the spacer and secure the flywheel with the fastening disc (Fig. 5, 3). Fully unscrew the fastening screw at the other end of the main axis to begin with.
- First slot the large counterweight, then the small set weight onto the other end of the main axle and screw the fastening screw back into the main axle (Fig. 5, 5,6,7). Move the counterweight and set weight to make the alignment of the main axis horizontal (pointer at 0 on the scale) and secure them both with fastening screws. You may need to make some fine adjustments with the screw.
- Attach the pointer for the flywheel to the flywheel itself with velcro as shown in Fig. 1 and Fig. 2.

SET-UP AND EXPERIMENT PROCEDURE

- Set-up the two photoelectric gates with the help of stand components such that they are positioned as in Fig. 1 and Fig. 2 and use the cable with the 8-pin miniDIN plugs to connect them to analog inputs A and B of the NET*log*™ unit.

Note:
When measuring nutation, the photo gate which records the period of nutation should not be operated in "internal light barrier mode" but in "laser light barrier mode". This is activated by closing the mechanical aperture and aligning the diode laser with the opening in the side of the photo gate.

- Turn on the computer and run 3B NET*lab*™. Connect the 3B NET*log* ™ unit to the computer with the USB cable. In 3B NET*lab* ™, click the "Test" under "Device connection", to check the connection.
- Configure analog inputs A and B to 20 V DC in 3B NET*lab*™ and click the "Inputs OK" button to confirm.
- Set the following parameters:

Measuring interval/Rate:	10 ms 100 Hz Osc
Number of measurement values:	10000
Duration of measurement:	01:40.0 s

Click the "Parameters OK" button to confirm.

- For measuring precession, suspend the weight holder with three weights on it from the hole in the front end of the main axle.
- For measuring nutation do not suspend any weights.
- Insert the metal sleeve of the starter thread into the hole in the bobbin and wind the starter thread around the bobbin.
- Press the “Start” button in 3B NET*lab*™ to start recording measurements.
- Grip the rear end of the main axle with one hand and hold the grip on the end of the starter thread with the other. Start the flywheel rotating by pulling fast and hard, but steadily, on the starter thread. Then immediately let go of the main axle.

Note:
When pulling the starter thread, keep it under tension until it has all rolled off the bobbin, otherwise it could get tangled in it.

It may be best to have two people doing the starting of 3B NET*lab* ™ and turning the flywheel.

Make sure that the gyroscope remains more or less horizontal while doing this.

- For measuring nutation, get the nutation started by hitting the gyroscope axle gently from the side.

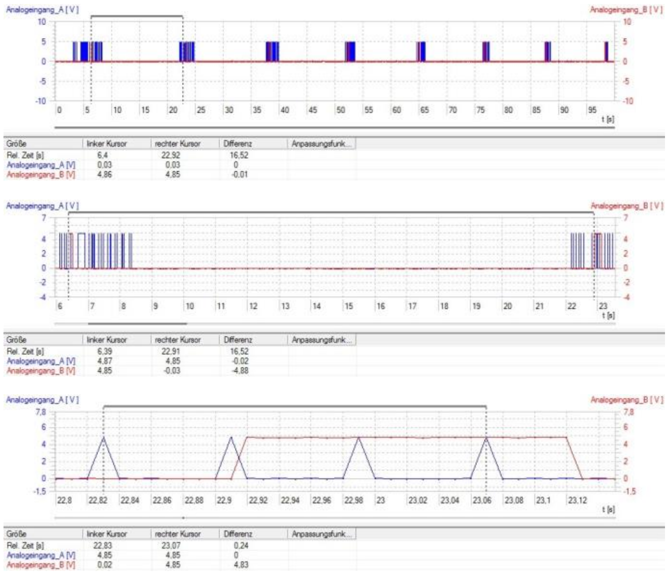

Fig. 6 Precession of gyroscope. Recorded timing diagram (top) and determination of precession period T_P (centre) and rotation period T_R (bottom) in 3B NET*lab* ™.

SAMPLE MEASUREMENTS AND EVALUATION

Mass *M* of flywheel:	1.5 kg
Radius *R* of flywheel:	12.5 cm
Mass of additional weight *m*:	150 g
Distance *r* from fulcrum of axle to point from which additional weight is suspended	22.5 cm

Fig. 6 (top) and Fig. 7 (top) show some typical results for precession and nutation measurements using 3B NET*lab* ™ and the set-ups shown in Fig. 1 and Fig. 2.

- Determine the periods for precession, nutation and rotation T_P, T_N and T_R via the changes in the timing diagram recorded for the pulses.

In Fig. 6 and Fig. 7 analog input A (blue) corresponds to the timing diagram for the rotation signal and analog input B (red) is the timing diagram for the precession or nutation signals.

The period of precession can be read off directly with the help of the cursors. It is the interval between two pulses. For example, the first precession period in Fig. 6 (centre) $T_P = 16.52$ s.

The period of nutation can also be read off directly with the help of the cursors, see Fig. 7 (centre). To do this, identify at least three successive and evenly spaced pulses in the signal. Since in one period of nutation, the photo gate will be obscured three times, the nutation period corresponds to the interval between the first pulse and the third one. For example, the first nutation period in Fig. 7 (centre) $T_N = 1.64$ s.

The rotation period is obtained from multiple successive pulses in the same regions where the corresponding precession or nutation periods were determined.

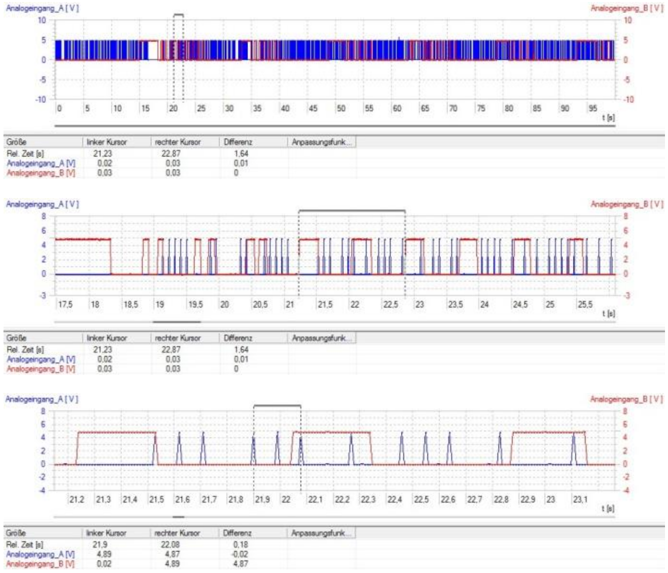

Fig. 7 Nutation of gyroscope. Recorded timing diagram (top) and determination of nutation period T_N (centre) and rotation period T_R (bottom) in 3B NET*lab* ™.

For example, the period of rotation corresponding to the first precession period in Fig. 6 (bottom) $T_R = 0.24$ s/3 = 0.08 s, and that corresponding to the first nutation period in Fig. 7 (bottom) $T_R = 0.18$ s/2 = 0.09 s.

- Enter the periods you have obtained in tables 1 and 2.

Table 1: Precession of gyroscope. Precession periods T_P and rotation periods T_R obtained by measurement and frequencies of rotation f_R calculated therefrom.

T_P / s	T_R / s	f_R / Hz
16.52	0.08	12.50
15.31	0.09	11.11
14.17	0.10	10.00
12.68	0.11	9.09
11.06	0.12	8.33
10.63	0.13	7.69

Table 2: Nutation of gyroscope. Nutation periods T_N and rotation periods T_R obtained by measurement.

T_N / s	T_R / s
1.64	0.09
1.78	0.10
1.99	0.11
2.19	0.12
2.35	0.13

- Calculate the frequencies of rotation from the rotation periods in Table 1 and enter these into Table 1 as well.
- Plot the rotation frequencies and precession periods in a graph and fit a straight line through the origin to the points (Fig. 8).
- Use equation (6) to calculate the moment of inertia of the flywheel from the slope of the graph a:

$$f_R = \frac{m \cdot g \cdot r}{4 \cdot \pi^2 \cdot I} \cdot T_P = a \cdot T_P \Rightarrow$$

$$I = \frac{m \cdot g \cdot r}{4 \cdot \pi^2 \cdot a} = \frac{0.15\,\text{kg} \cdot 9.81\,\frac{\text{m}}{\text{s}} \cdot 0.225\,\text{m}}{4 \cdot \pi^2 \cdot 0.73\,\frac{1}{\text{s}^2}} .$$

$$= 0.0115\,\text{kg} \cdot \text{m}^2$$

- Determine the moment of inertia of the flywheel by means of equation (2):

$$I = \frac{1}{2} \cdot 1.5\,\text{kg} \cdot (0.125\,\text{m})^2 = 0.0117\,\text{kg} \cdot \text{m}^2 .$$

The values are in agreement within the bounds of their relative measurement errors of about 1.5%.

- Plot the periods of rotation Tab. 2 against the periods of rotation in a graph and fit a straight line through the origin to the points (Fig. 9).

Direct proportionality between the periods of nutation and rotation as predicted by equation (7) is thereby confirmed.

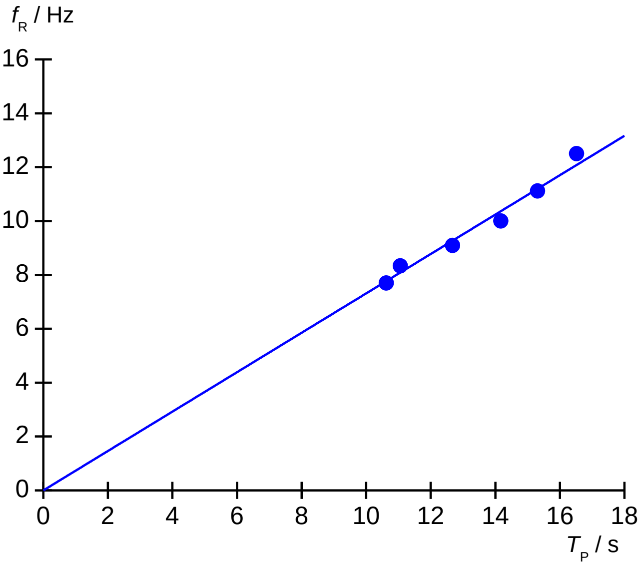

Fig. 8 Frequency of rotation f_R of a rotating disc as a function of the period of precession T_P

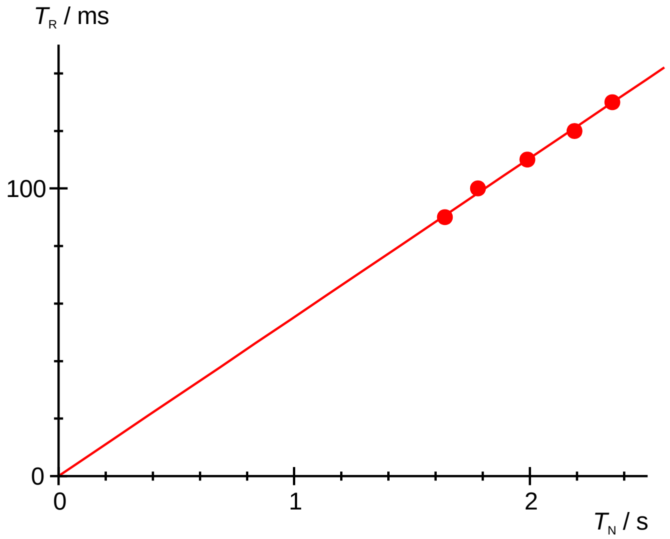

Fig. 9 Period of rotation T_R as a function of period of nutation T_N.