Acoustics

Wavelength and speed of sound

PHYSICS EXPERIMENT

Speed of Sound in Air

GENERATE AND MEASURE STANDING SOUND WAVES IN KUNDT'S TUBE.

- **Generate standing waves in Kundt's tube with both ends closed off.**
- **Measure the fundamental frequency as a function of the length of the Kundt's tube.**
- **Measure the frequencies of the fundamental and overtones for a fixed length of tube.**
- **Determine the speed of propagation of the wave from the resonant frequencies.**

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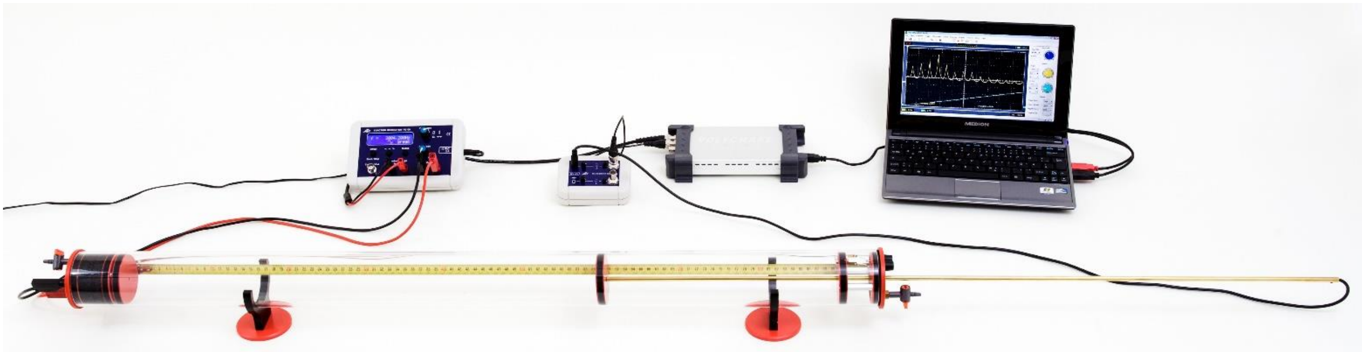

Fig. 1: Measurement set-up.

GENERAL PRINCIPLES

It is possible to generate standing waves in Kundt's tube by producing waves of a suitable resonant frequency from a loudspeaker at one end of the tube, which are then reflected by the cap at the other end. If the length of the tube is known, it is possible to determine the speed of propagation of the waves from the resonant frequency and the number of the harmonics.

Sound waves propagate in air and other gases by means of rapid changes in pressure and density. It is easiest to describe them on the basis of the sound pressure, which is superimposed on top of atmospheric pressure. As an alternative to the sound pressure p, the sound velocity v can also be used to describe a sound wave. That is the average velocity of gas molecules at a given point x in the oscillating medium at a point in time t. Pressure and velocity of sound are linked, for example by an Euler equation of motion:

$$(1)\quad -\frac{\partial p}{\partial x} = \rho_0 \cdot \frac{\partial v}{\partial t}$$

ρ_0: density of gas

In Kundt's tube, sound waves propagate along the length of the tube, i.e. they can be described with the help of a one-dimensional wave equation, which applies to both sound pressure and velocity:

$$(2)\quad \frac{\partial^2 p(x,t)}{\partial t^2} = c^2 \cdot \frac{\partial^2 p(x,t)}{\partial x^2} \quad \text{or}$$

$$\frac{\partial^2 v(x,t)}{\partial t^2} = c^2 \cdot \frac{\partial^2 v(x,t)}{\partial x^2}$$

c: speed of sound

This experiment studies harmonic waves, which are reflected at the end of the Kundt's tube.

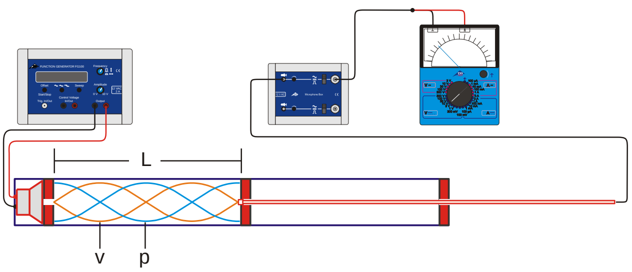

Fig. 2: Schematic of experiment set-up

To find the solutions to the wave equation, the superposition of the outgoing and reflected waves needs to be taken into account:

$$(3)\quad p = p_{0>}\cdot e^{2\pi i\left(f\cdot t-\frac{x}{\lambda}\right)} + p_{0<}\cdot e^{2\pi i\left(f\cdot t+\frac{x}{\lambda}\right)}$$

$$v = v_{0>}\cdot e^{2\pi i\left(f\cdot t-\frac{x}{\lambda}\right)} + v_{0<}\cdot e^{2\pi i\left(f\cdot t+\frac{x}{\lambda}\right)}$$

$p_{0>}$, $v_{0>}$: amplitudes of outgoing wave,
$p_{0<}$, $v_{0<}$: amplitudes of returning wave
f: frequency, λ: wavelength

In this case

$$(4)\quad f\cdot\lambda = c$$

By substituting these solutions into equation (1) and considering the outgoing and returning waves separately, the following can be derived:

$$(5)\quad p_{0>} = v_{0>}\cdot Z \text{ or } p_{0<} = v_{0<}\cdot Z\,.$$

The quantity

$$(6)\quad Z = c\cdot\rho_0$$

is known as the sound impedance and corresponds to the resistance to the waves from the medium itself. It plays a key role in considerations of the reflection of a sound wave by walls with an impedance of W.

The following then applies:

$$(7)\quad r_v = \frac{v_{0<}}{v_{0>}} = \frac{Z-W}{Z+W} \text{ and } r_p = \frac{p_{0<}}{p_{0>}} = \frac{\frac{1}{Z}-\frac{1}{W}}{\frac{1}{Z}+\frac{1}{W}}$$

In this experiment W is much higher than Z so that we may assume $r_v = 1$ and $r_p = -1$.

If the reflecting wall is selected, for simplicity's sake, to be at $x = 0$, the spatial component of the sound wave can be derived from equation (3) as follows:

$$(8)\quad \begin{aligned} p &= p_{0>}\cdot\left(e^{-2\pi\cdot i\cdot\frac{x}{\lambda}} + e^{+2\pi\cdot i\cdot\frac{x}{\lambda}}\right)\cdot e^{-2\pi\cdot i\cdot f\cdot t} \\ &= 2\cdot p_{0>}\cdot\cos\left(\frac{2\pi}{\lambda}\cdot x\right)\cdot e^{-2\pi\cdot i\cdot f\cdot t} \end{aligned}$$

and

$$\begin{aligned} v &= v_{0>}\cdot\left(e^{-2\pi\cdot i\cdot\frac{x}{\lambda}} - e^{+2\pi\cdot i\cdot\frac{x}{\lambda}}\right)\cdot e^{-2\pi\cdot i\cdot f\cdot t} \\ &= -2\cdot i\cdot v_{0>}\cdot\sin\left(\frac{2\pi}{\lambda}\cdot x\right)\cdot e^{-2\pi\cdot i\cdot f\cdot t} \end{aligned}$$

Only the real components of these terms have any actual physical relevance. They correspond to standing sound waves which have a pressure anti-node at the end wall (i.e. at $x = 0$), while the sound velocity at that point has a node in its oscillation. The velocity is phase shifted ahead of the pressure by 90°.

Sound waves are generated by a loudspeaker at a distance L from the wall. These waves oscillate with frequency f. At this point, too the pressure has an anti-node and the velocity has a node. Such boundary conditions are only fulfilled when L is an integer multiple of half the wavelength:

$$(9)\quad L = n\cdot\frac{\lambda_n}{2}$$

From equation (3) then, the frequencies must fulfil the following condition for resonance:

$$(10)\quad f_n = n\cdot\frac{c}{2\cdot L}\,.$$

LIST OF EQUIPMENT

1	Kundt's tube E	1017339 (U8498308)
1	Microphone probe, long	1017342 (U8498282)
1	Microphone box @230V	1014520 (U8498283-230)
or		
1	Microphone box @115V	1014521 (U8498283-115)
1	Function generator FG 100 @230V	1009957 (U8533600-230)
or		
1	Function generator FG 100 @115V	1009956 (U8533600-115)
1	USB oscilloscope 2x50 MHz	1017264 (U112491)
1	Escola 100 analog multimeter	1013527 (U8557380
2	HF patch cords, BNC/4-mm plugs	1002748 (U11257)
1	Pair of safety experiment leads, 75 cm	1002849 (U13812)
1	HF patch cord	1002746 (U11255)

SET UP AND PROCEDURE

- Set up the sound tube on the supplied feet (fig. 1).
- First insert the capillary disc into the tube and then the end cap with the sockets for the speaker. Set them up such that they meet flush inside the tube.
- Insert the long microphone probe through the hole in the end cap with the heating rod sockets as far as it will go and then push it through the guide disc.
- Screw the probe disc onto the long microphone probe.
- Insert the end cap with the heater sockets and with the long microphone probe attached into the tube.

Note:
If necessary apply some glycerine or soap to the sealing gaskets to make insertion easier.

- Clamp the movable scale into its holder on the base unit and move it until the zero mark coincides with the front edge of the capillary disc.
- Connect the output sockets of the function generator to the speaker sockets.
- Connect the long microphone probe to the Channel A input of the microphone box.
- Connect the channel A output of the microphone box to the analog multimeter by means of a BNC/4-mm cable.
- Connect the plug-in power supply to the microphone box and plug it into the mains.

In this experiment the frequency *f* of the signal to the speaker will be continuously varied while the microphone probe measures the sound pressure at the reflecting end wall. Resonance occurs when the microphone signal reaches its maximum amplitude.

Fundamental frequency as a function of length

- By moving the microphone probe in or out, set up a length L = 80 cm between the capillary disc and the probe disc.
- Match up the amplitude of the function generator and the measuring range of the multimeter as appropriate, even during the experiment if necessary. Take note of the maximum power limit for the speaker (U_{rms} = 6 V max.).
- Set the frequency to 150 Hz and gradually increase it. Find the frequency at which the multimeter displays the *first maximum* and enter the value into Table 1.

The frequency f_1 found in this way corresponds to that of the fundamental oscillation when L = 80 cm.

- Adjust the length L between the capillary disc and the probe disc in steps of 10 cm down to L = 10 cm and repeat the measurement for each of these steps. In each case, use the resonant frequency f_1 found for the previous step as the new start frequency.

Frequencies of fundamental and harmonics for a fixed length

- By moving the microphone probe in or out, set up a length L = 60 cm between the capillary disc and the probe disc.
- Match up the amplitude of the function generator and the measuring range of the multimeter as appropriate, even during the experiment if necessary. Take note of the maximum power limit for the speaker (U_{rms} = 6 V max.).
- Slowly increase the frequency from 250 Hz to 3500 Hz. Enter the frequencies where the multimeter displays maximum values into table 2.

Frequency scan

- By moving the microphone probe in or out, set up a length L = 60 cm between the capillary disc and the probe disc.
- Use an HF cable to connect the channel A output of the microphone box to channel CH1 of the USB oscilloscope.
- Connect the pair of sockets labelled “Control voltage input/Ramp output” on the function generator directly to channel CH2 of the USB oscilloscope.
- Press the “Sweep” on the function generator and configure the following parameters:

Freq. start	100 Hz
stop	6000 Hz
int. continuous mode	
Time	4.00 sec

- Set the marker for the horizontal trigger position on the USB oscilloscope all the way to the left and move the y position marker for CH 1 down by half a division from the middle and that for CH 2 right to the bottom, then configure the following parameters:

Time/div	400 ms
CH1	1.00 V DC
CH2	2.00 V DC
Trigger Mode	Edge
Sweep	Auto
Source	CH2
Slope	–
Level	850 mV

- Start the function generator sweep and wait until the full frequency spectrum is displayed on the USB oscilloscope.
- Stop the measurement by pressing the “Stop” button on the USB oscilloscope and take a photograph of the trace on the screen.

SAMPLE MEASUREMENT AND EVALUATION

Table 1: Resonant frequencies of fundamental oscillation (n = 1) as measured and wavelength as calculated using equation (9) for various lengths L.

L / m	f_1 / Hz	$\lambda_1 = 2 \cdot L$
0.8	218	1.60 m
0.7	250	1.40 m
0.6	292	1.20 m
0.5	350	1.00 m
0.4	436	0.80 m
0.3	583	0.60 m
0.2	884	0.40 m
0.1	1768	0.20 m

Table 2: Resonant frequencies of fundamental oscillation and harmonics as measured and wavelength as calculated using equation (9) for a fixed length L = 0.6 m.

n	f_n / Hz	$\lambda_n = \frac{2 \cdot L}{n}$
1	289	1.20 m
2	582	0.60 m
3	876	0.40 m
4	1164	0.30 m
5	1453	0.24 m
6	1746	0.20 m
7	2039	0.17 m
8	2331	0.15 m
9	2623	0.13 m
10	3206	0.12 m
11	3494	0.10 m

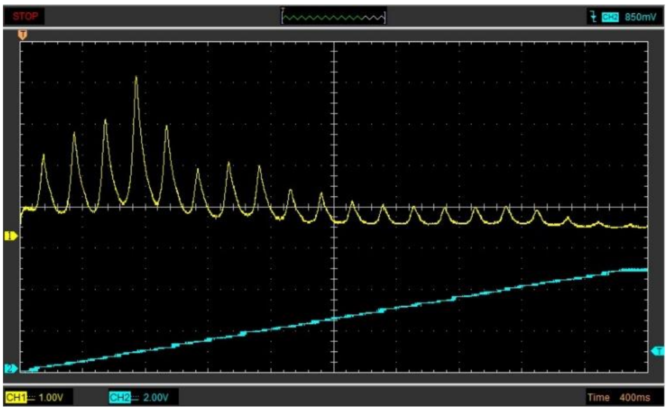

Fig. 3: Oscilloscope trace of frequency scan, L = 0.6 m.

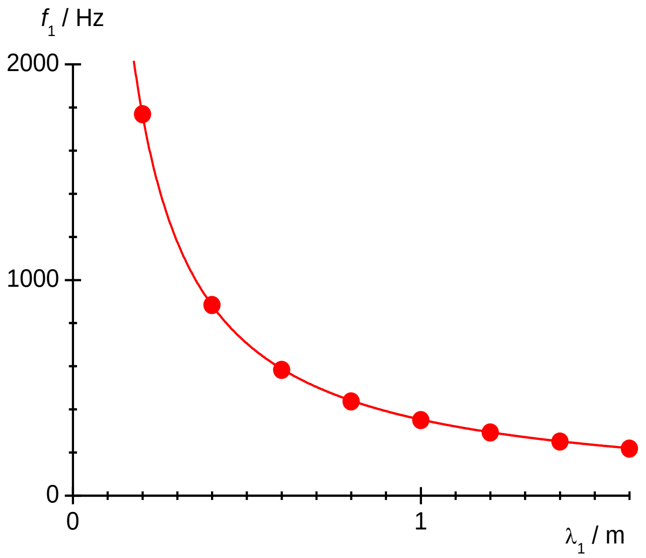

Fig. 4: Graph of frequency against wavelength for fundamental frequencies (n = 1) with variable length L.

Fundamental frequency as a function of length

According to equation (9) the wavelengths of the resonances f_1 for the fundamental wave are as follows:

(11) $\lambda_1 = 2 \cdot L$.

- Calculate wavelengths using equation (11) and enter them into Table 1.
- Plot the resonance frequencies measured and the wavelengths calculated (Table 2) in a graph of f against λ (Fig. 4).
- Fit a hyperbola to the measurement points:

(12) $f_1 = a \cdot \frac{1}{\lambda_1}$.

The fitting of the curve confirms equation (4). The speed of sound in air is equal to the gradient a:

(13) $c = a = 353 \frac{\text{m}}{\text{s}}$.

The value differs by about 2% from that quoted in literature c = 346 m/s at T = 25°C.

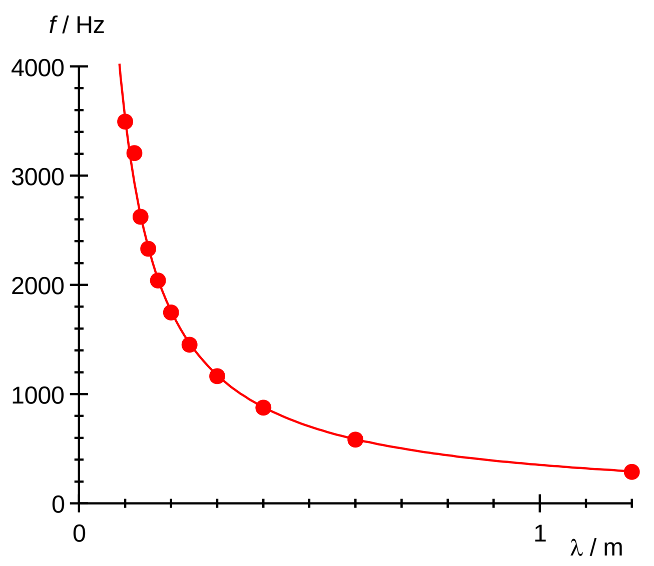

Fig. 5: Graph of frequency against wavelength for fundamental and harmonics when L = 0.6 m.

Frequencies of fundamental and harmonics for a fixed length

According to equation (9), the resonant frequencies determined f_n should have wavelengths

$$(14)\ \lambda_n = \frac{2 \cdot L}{n}.$$

- Calculate the wavelengths using equation (14) and enter them into Table 2
- Plot the resonant frequencies measured against the calculated wavelengths (Table 2) in a graph (Fig. 5).
- Fit a hyperbola to the measurement points:

$$(15)\ f = a \cdot \frac{1}{\lambda}.$$

The fitting of the curve confirms equation (4). The speed of sound in air is equal to the gradient a:

$$(16)\ c = a = 356\,\frac{\text{m}}{\text{s}}.$$

The result differs by about 3% from the value quoted in literature c = 346 m/s at T = 25°C.

Frequency scan

The times t_n where the maxima in the frequency scan occur, can be determined with the help of the cursors of the USB oscilloscope. These times do not match the periods of oscillation T_n which correspond to the resonant frequencies f_n. The resonant frequencies need to be calculated from the frequency scan parameters as follows:

$$(17)\ f_n = f_{start} + \frac{t_n}{T_{sweep}} \cdot \left(f_{stop} - f_{start}\right) = 100\ Hz + \frac{t_n}{4\ s} \cdot 5900\ Hz.$$

From this the speed of sound can again be determined by the means described above.