Electricity

DC and AC Circuits

Impedance of a Coil in an AC Circuit

DETERMINE INDUCTIVE IMPEDANCE AS A FUNCTION OF INDUCTANCE AND FREQUENCY

- **Determine the amplitude and phase of inductive impedance as a function of the inductance.**
- **Determine the amplitude and phase of inductive impedance as a function of the frequency.**

UE3050211

04/18 UD

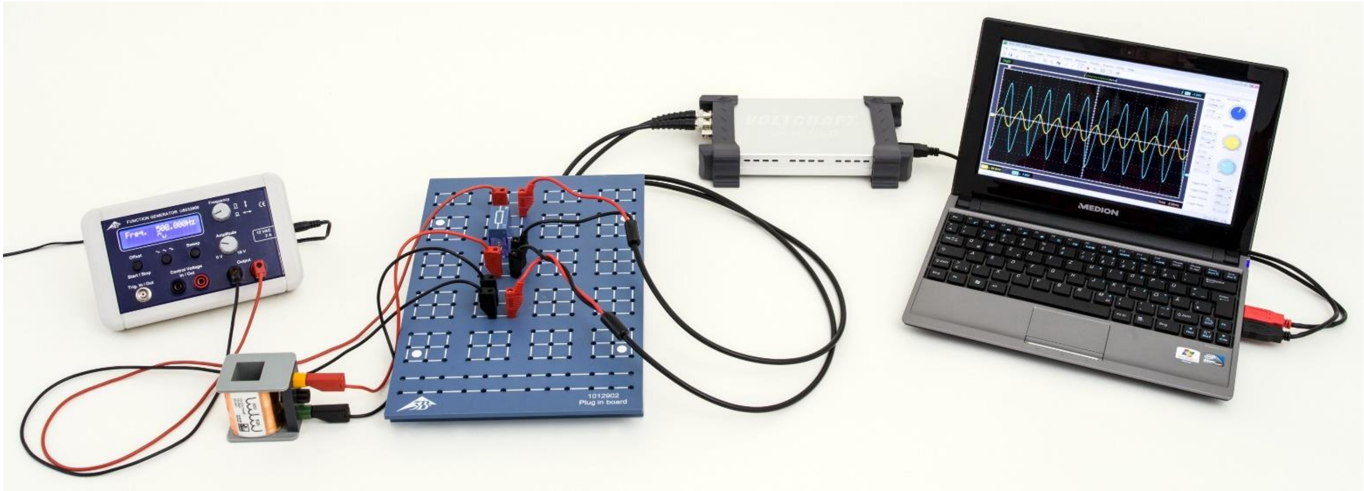

Fig. 1: Experiment set-up

GENERAL PRINCIPLES

Any change in the current through a coil induces a voltage which acts such as to oppose the change in current. If an alternating current flows, an AC voltage will be induced, which is shifted in phase with respect to the current. In mathematical terms, the relationship can be expressed most easily if current, voltage and impedance are regarded as complex values, whereby the real components need to be considered.

The relationship between current and voltage for a coil is as follows:

(1) $U = L \cdot \frac{\mathrm{d}I}{\mathrm{d}t}$.

I: Current, U: Voltage, L: Inductance

Assume the following voltage is applied:

(2) $U = U_0 \cdot \exp(i\omega t)$

This gives rise to a current as follows:

(3) $I = \frac{U_0}{i \cdot \omega \cdot L} \cdot \exp(i\omega t)$.

The impedance associated with the inductor L can then be defined as in the following equation:

(4) $X_\mathrm{L} = \frac{U}{I} = i \cdot \omega \cdot L = i \cdot 2\pi \cdot f \cdot L$.

The real component of this is measurable, therefore

(5) $U = U_0 \cdot \cos \omega t$

(6) $I = \frac{U_0}{\omega \cdot L} \cos\left(\omega t - \frac{\pi}{2}\right) = I_0 \cos\left(\omega t - \frac{\pi}{2}\right)$

(7) $X_\mathrm{L} = \frac{U_0}{I_0} = \omega \cdot L = 2\pi \cdot f \cdot L$

In this experiment, a frequency generator supplies an alternating voltage with a frequency of up to 2 kHz. A dual-channel oscilloscope is used to record the voltage and current, so that the amplitude and phase of both can be determined. The current through the capacitor is related to the voltage drop across a resistor *R* with a value which is negligible in comparison to the inductive impedance exhibited by the coil itself.

As an option, voltage and current can also be recorded using the VinciLab data logger and Coach 7 software with voltage sensors.

LIST OF EQUIPMENT

1	Plug-In Board for Components	1012902 (U33250)
2	Coil S with 1200 Turns	1001002 (U8498085)
1	Resistor 10 Ω, 2 W, P2W19	1012904 (U333012)
1	Function Generator FG 100 @230V	1009957 (U8533600-230)
or		
1	Function Generator FG 100 @115V	1009956 (U8533600-115)
1	PC Oscilloscope 2x25 MHz	1020857 (U11830)
2	HF Patch Cord, BNC/4 mm Plug	1002748 (U11257)
1	Set of 15 Experiment Leads	1002840 (U13800)
Optional		
1	VinciLab	1021477 (UCMA-001)
1	Coach 7, School Site License 5 Years	1021522 (UCMA-18500)
or		
1	Coach 7, University License 5 Years	1021524 (UCMA-185U)
2	Voltage Sensor 10 V, Differential	1021680 (UCMA-0210i)
1	Voltage Sensor 500 mV, Differential	1021681 (UCMA-BT32i)
1	Sensor Cable	1021514 (UCMA-BTsc1)

SET-UP AND EXPERIMENT PROCEDURE

- Set up the measuring equipment (Fig. 1) as shown in the circuit diagram (Fig. 2) with resistor R =10 Ω and a 1200-turn coil (L = 23 mH, R_L = 19 Ω).
- Connect the measurement lead for the recording of the voltage curve $U_R(t) = R \cdot I(t)$ across the resistor to channel CH1 and the line for recording the curve $U_L(t)$ across the coil to channel CH2 of the oscilloscope.
- Set the following parameters on the PC oscilloscope:

  Horizontal:

Time-base:	500 µs/div
Horizontal trigger position:	0.0 ns

  Vertical:
  CH1:

Voltage scale division:	200 mV/div DC
Zero position:	0.0 divs

  CH2:

Voltage scale division:	1 V/div DC
Zero position:	0.0 divs

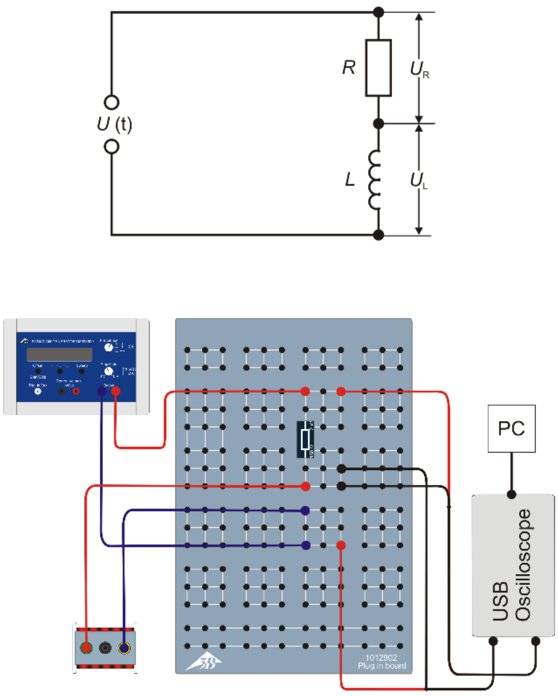

Fig. 2: Circuit diagram sketch (top) and set-up schematic (bottom).

  Trigger:
  Single (not Alternate)

Source:	CH2
Mode:	Edge
Edge:	Rising
Threshold:	0.000 mV
Trigger mode:	Auto

  It may be necessary to change the Time/div and Volts/div settings during the series of measurements to ensure the signal is optimally displayed.

- Set a frequency f = 500 Hz.
- Select a sinusoidal wave form on the function generator and adjust the amplitude of the input signal to U_0 = 4 V. Set the amplitude control in such a way that the maximum and minimum of the sinusoidal signal on channel CH2 of the oscilloscope are separated by four divisions (for a setting of 1 V/div).

The value of the resistor R is negligible in comparison to the impedance of the inductor X_L at the frequencies being observed, but the ohmic resistance R_L of the coil does need to be explicitly taken into account.

Phase shift between current and voltage

- Observe and make a note of the relative positions of the voltage curves $U_L(t)$ and $U_R(t)$ across the coil and resistor.

How the inductive impedance depends on the inductance

- Using two coils with 1200 turns (L = 23 mH, R_L = 19 Ω), vary the number of turns being tapped in order to obtain the inductance values listed in Table 1. For each setting read off the amplitudes U_{L0} and U_{R0} from the scope and enter them into Table 1 as well.

Inductance values for N = 400 and 800 (as tapped from the coil) can be calculated using the following formula:

$$\text{(8a)}\quad \frac{L}{23\,\text{mH}} = \left(\frac{N}{1200}\right)^2 \Leftrightarrow L = \left(\frac{N}{1200}\right)^2 \cdot 23\,\text{mH},$$

Values for N = 1600, 2000 and 2400 (with two coils in series) are obtained as follows:

$$\text{(8b)}\quad L = L_{1200} + L_{\text{N-1200}} = 23\,\text{mH} + L_{\text{N-1200}}$$

$L_{\text{N-1200}}$: Inductance of coil with N-1200 turns

The corresponding ohmic resistance values R_L can be calculated as follows:

$$\text{(9)}\quad \frac{R_L}{19\,\Omega} = \frac{N}{1200} \Leftrightarrow R_L = \frac{N}{1200} \cdot 19\,\Omega.$$

How the inductive impedance depends on the frequency

- Use one coil with 1200 turns (L = 23 mH, R_L = 19 Ω) and the 10 Ω resistor for the measurement.
- Set up the frequencies listed in Table 2 on the function generator one by one, read off amplitudes U_{L0} and U_{R0} from the oscilloscope and enter them into Table 2 as well.

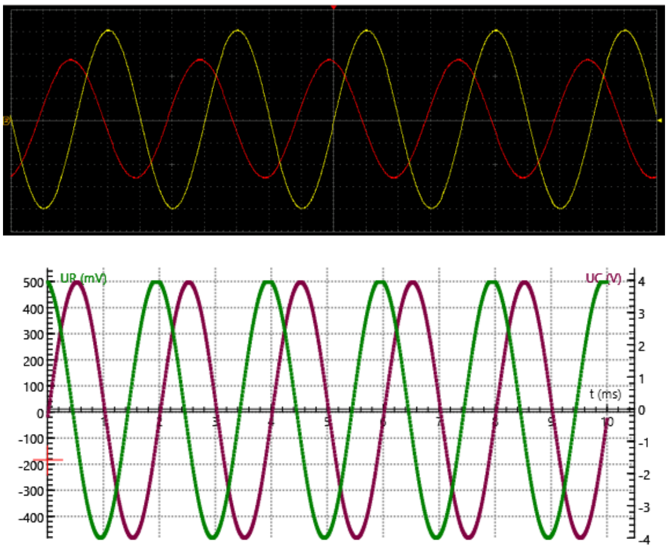

Fig. 3: Coil in an AC circuit: Current and voltage over time. Top: Recording using PC oscilloscope (current: red, voltage: yellow). Bottom: Recording using VinciLab/Coach7 (current: green, voltage: violet).

SAMPLE MEASUREMENT AND EVALUATION

Phase shift between current and voltage

The current signal is shifted by a quarter of the period to the left with respect to the voltage signal (Fig. 3).

The current through the coil lags 90° behind the voltage across the coil because any change in current induces back emf.

How the inductive impedance depends on the inductance and frequency

- Calculate the amplitude of the current through the coil using the following formula:

$$\text{(10)}\quad I_0 = \frac{U_{R0}}{R} = \frac{U_{R0}}{10\,\Omega}$$

Enter the results into Table 1.

- Calculate the total resistance of the coil using the formula below:

$$\text{(11)}\quad Z_L = \sqrt{R_L^2 + X_L^2} = \frac{U_{L0}}{I_0}$$

Enter the results into Table 1.

- Calculate the inductive impedance using the following formula

$$\text{(12)}\quad X_L = \sqrt{Z^2 - R_L^2}$$

Enter the results into Table 1.

- Plot the inductive impedance values X_L against inductance (Table 1, Fig. 4) and frequency (Table 2, Fig. 5).

As per equation (4), the inductive impedance X_L is proportional to the frequency f and the inductance L. In the relevant graphs, the measurements therefore lie along a straight line through the origin within the measurement tolerances.

Tab. 1: How inductive impedance depends on inductance, f = 500 Hz, R = 10 Ω, U_0 = 4 V.

N	L mH	R_L Ω	U_{L0} V	U_{R0} mV	I_0 mA	Z_L Ω	X_L Ω
400	2.6	6.3	2.063	2220	222.0	9.3	6.8
800	10.2	12.7	3.475	860	86.0	40.4	38.4
1200	23.0	19.0	3.725	470	47.0	79.3	77.0
1600	25.6	25.3	3.850	453	45.3	85.0	81.1
2000	33.2	31.7	3.750	313	31.3	119.8	115.5
2400	46.0	38.0	3.775	234	23.4	161.3	156.8

Tab. 2: How inductive impedance depends on frequency, L = 23 mH, R_L = 19 Ω, R = 10 Ω, U_0 = 4 V.

f Hz	U_{L0} V	U_{R0} mV	I_0 mA	Z_L Ω	X_L Ω
100	2.850	995	99.5	28.6	21.4
300	3.525	725	72.5	48.6	44.7
500	3.725	488	48.8	76.3	73.9
800	3.800	325	32.5	116.9	115.3
1200	3.825	217	21.7	176.3	175.3
2000	3.875	131	13.1	295.8	295.2

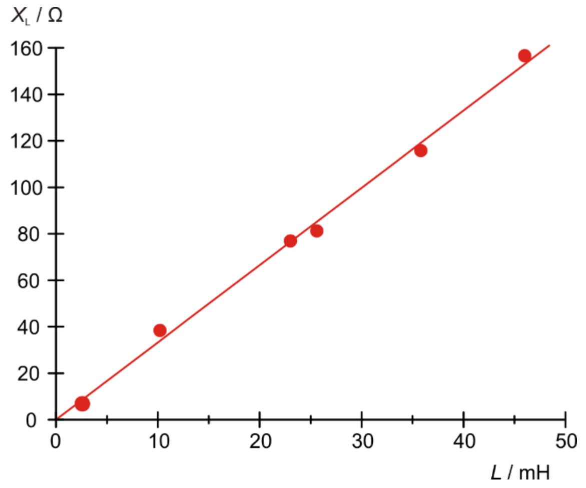

Fig. 4: Inductive impedance X_L as a function of inductance L.

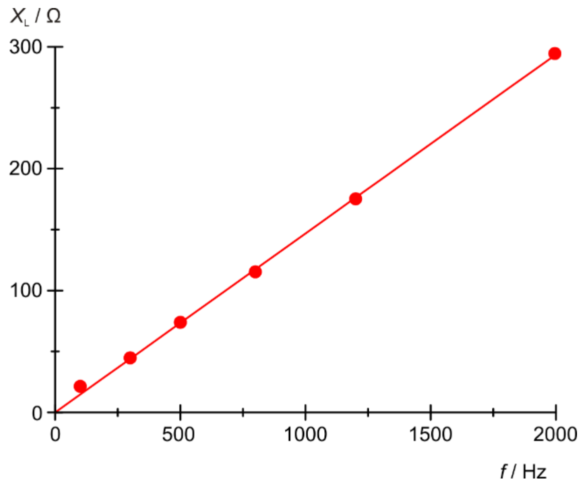

Fig. 5: Inductive impedance X_L as a function of frequency f.