Electricity

DC and AC

AC Resistance

DETERMINE THE AC RESISTANCE IN A CIRCUIT WITH INDUCTIVE AND RESISTIVE LOADS.

- **Determine the amplitude and phase of the overall resistance as a function of frequency for a series circuit.**
- **Determine the amplitude and phase of the overall resistance as a function of frequency for a parallel circuit.**
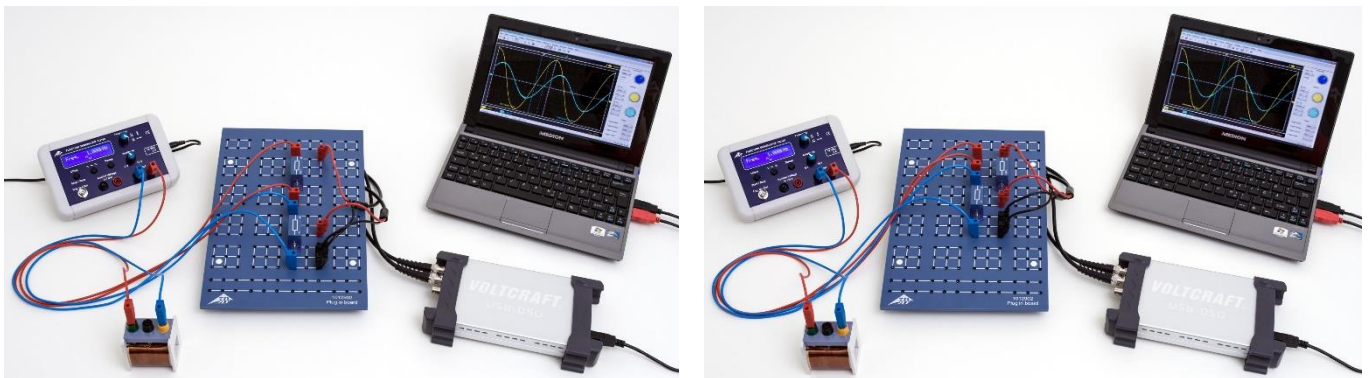
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Fig. 1: Measurement set-up for series circuit (left) and parallel circuit (right).

BASIC PRINCIPLES

In AC circuits, it is common to use complex numbers to describe the resistance in circuits with inductors because this actually makes calculation easier. This is because not only the amplitude of the current and voltage is a factor, but also the phase relationships between the two need to be taken into account (this complex resistance is usually called impedance). Series and parallel circuits with both ohmic and inductive resistance can then be described quite easily, although in each case, only the real component is measurable).

The complex resistance (impedance) of a coil of inductance L in a circuit with an alternating current of frequency f is as follows:

(1) $\quad X_L = i \cdot X_{L0} = i \cdot \omega \cdot L$
Angular frequency $\omega = 2 \cdot \pi \cdot f$.

Therefore the total resistance (impedance) of a series circuit containing a coil with a resistance R is

(2) $\quad Z_S = i \cdot 2 \cdot \pi \cdot f \cdot L + R$.

For a parallel circuit, the total resistance can be assigned thus:

(3) $\quad Z_P = \dfrac{1}{\dfrac{1}{i \cdot 2 \cdot \pi \cdot f \cdot L} + \dfrac{1}{R}}$

The usual way of writing this is as follows:

(4) $\quad Z = Z_0 \cdot \exp(i \cdot \varphi)$.

This becomes

(5) $\quad Z_S = Z_{S0} \cdot \exp\left(i \cdot \varphi_S\right) = \sqrt{\left(2 \cdot \pi \cdot f \cdot L\right)^2 + R^2} \cdot \exp\left(i \cdot \varphi_S\right)$
where $\tan\varphi_S = \dfrac{2 \cdot \pi \cdot f \cdot L}{R}$

and

(6) $\quad Z_P = Z_{P0} \cdot \exp\left(i \cdot \varphi_P\right) = \dfrac{2 \cdot \pi \cdot f \cdot L \cdot R}{\sqrt{\left(2 \cdot \pi \cdot f \cdot L\right)^2 + R^2}} \cdot \exp\left(i \cdot \varphi_P\right)$

where $\tan\varphi_P = \dfrac{R}{2 \cdot \pi \cdot f \cdot L}$.

If for each of the total impedance values $Z = Z_S$ and Z_P the following voltage is applied:

$$(7) \quad U = U_0 \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t)$$

this will result in the following current flowing:

$$(8) \quad \begin{aligned} I &= \frac{U_0}{Z_0} \cdot \exp(i \cdot (2 \cdot \pi \cdot f \cdot t - \varphi)) \\ &= I_0 \cdot \exp(i \cdot (2 \cdot \pi \cdot f \cdot t - \varphi)) \end{aligned}.$$

In this experiment this will be measured by finding the drop in voltage $U_m(t)$ across a load resistor R_m (Fig. 2, 3), which is of such a value that $U_{m0} << U_0$, i.e. nearly all of the voltage drops across Z_S or Z_P. The current obtained in this way flows through Z_S and through Z_P since both these resistors are in series with R_m (see equivalent circuit diagram in Fig. 2, 3). Since $U_m(t) = I(t) \cdot R_m$, the change in voltage over time $U_m(t)$ will also be reflected in the change in current over time $I(t)$.

LIST OF EQUIPMENT

1 Plug-In Board for Components	1012902 (U33250)
1 Resistor 1 Ω, 2 W, P2W19	1012903 (U333011)
1 Resistor 100 Ω, 2 W, P2W19	1012910 (U333018)
1 Function Generator FG 100 @230V	1009957 (U8533600-230)
or	
@115V	1009956 (U8533600-115)
1 PC Oscilloscope 2x25 MHz	1020857 (U11830)
2 HF Patch Cord, BNC/4 mm Plug	1002748 (U11257)
1 Set of 15 Experiment Leads, 75 cm 1 mm²	1002840 (U13800)
1 Coil S with 800 Taps	1001001 (U8498080)
1 Coil S with 1200 Taps	1001002 (U8498085)

SET UP AND PROCEDURE

Series circuit

- Assemble a set-up for measurement of the series circuit (as in Fig. 1, left) in accordance with the sketch of the circuit diagram (Fig. 2) in which $R_m = 10\,\Omega$, $C = 1\,\mu F$ and the coil has $N = 1200$ turns and inductance $L = 23.0$ mH.

- Connect the output signal $U_m(t) = I(t) \cdot R_m$ to channel 1 (CH1) of the oscilloscope and the input signal $U(t)$ to channel 2 (CH2).

- Configure the following starting parameters for the PC oscilloscope:

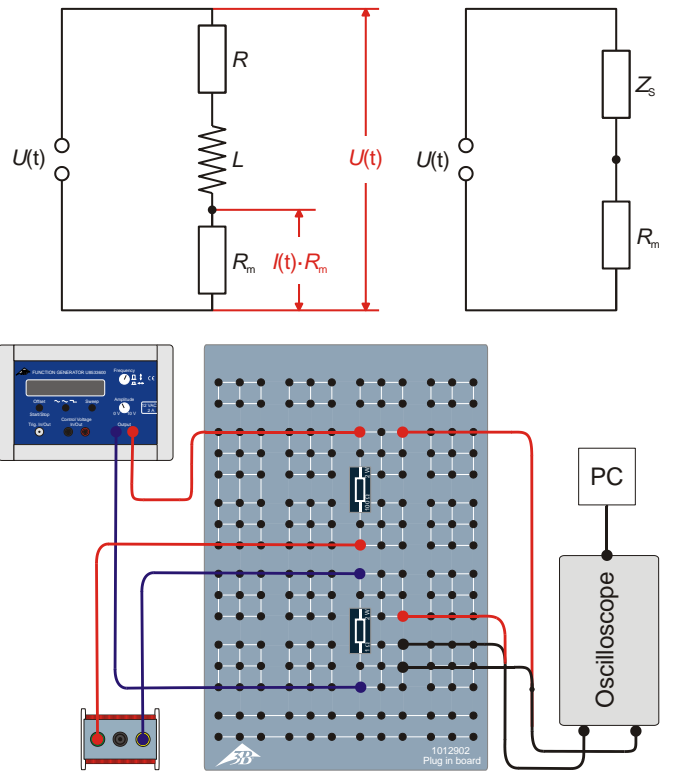|	
Time/DIV	25 μs
Volts/DIV CH1	2.00 mV AC
Volts/DIV CH2	2.00 V AC
Trigger Mode	Auto
Sweep	Edge
Source	CH2
Slope	+
Level	0.00 μV

Fig. 2: Circuit diagram (top left), equivalent circuit (top right) and schematic sketch of set-up (below) for series circuit.
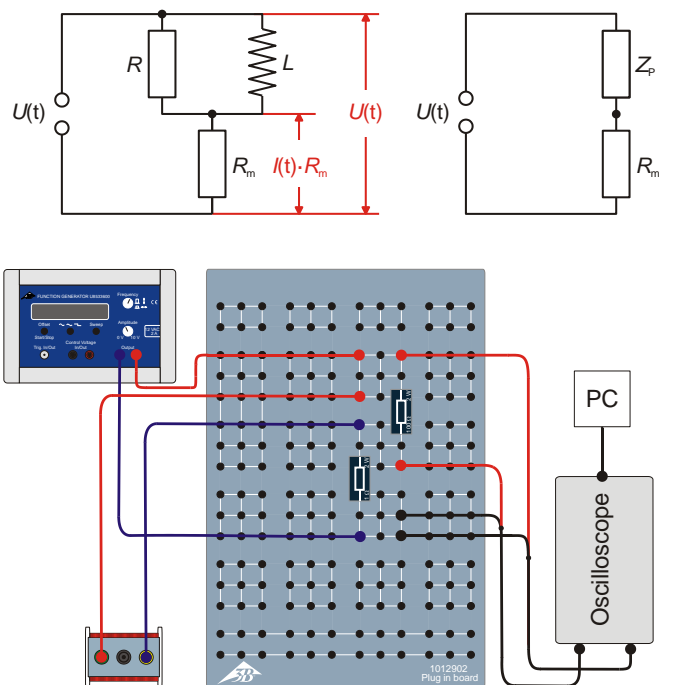

Fig. 3: Circuit diagram (top left), equivalent circuit (top right) and schematic sketch of set-up (below) for parallel circuit..

Note

The parameters Time/DIV und Volts/DIV for CH1 will need to be modified during the course of this sequence of measurements.

- Select Sine as the signal form for the function generator and set the amplitude of the input signal to $U_0 = 6$ V. Adjust the amplitude knob in such a way that the maximum or minimum of the sine-wave signal on channel CH2 of the scope (with 2 V per division) coincides with exactly 3 divisions.

- Set up frequencies of 10000 Hz, 5000 Hz, 2000 Hz, 1000 Hz, 500 Hz, 200 Hz and 50 Hz in sequence on the function generator. Calculate the corresponding periods of oscillation by means of the equation $T = 1 / f$ and enter the results into Table 1 along with the frequencies.

- Read off the amplitude U_{m0} of the output signal $U_m(t)$ from the oscilloscope and enter the results into Table 1.

- Read off the time differences Δt between zero crossings of the signals $U(t)$ and $U_m(t)$ from the oscilloscope and enter the values into Table 1.

- Repeat the measurements for all the same frequencies using the coil with $N = 800$ turns and inductance $L = 10.4$ mH. Enter these values into Table 1.

Parallel circuit

- Assemble a set-up for measurement of the parallel circuit (as in Fig. 1) in accordance with the sketch of the circuit diagram (Fig. 3) in which $R_m = 10$ Ω, $C = 1$ μF and the coil has $N = 1200$ turns and inductance $L = 23.0$ mH.

- Carry out the measurements in the same way as for the series circuit. Use the same initial parameters for the USB oscilloscope but set Volts/DIV for CH1 to 20.0 V AC

- Enter all the measurement values into Table 2.

SAMPLE MEASUREMENT

Tab. 1: Measurement values for series circuit.

f / Hz	T / ms	U_{m0} / mV		Δt / μs	
		$N = 1200$ $L = 23.0$ mH	$N = 800$ $L = 10.4$ mH	$N = 1200$ $L = 23.0$ mH	$N = 800$ $L = 10.4$ mH
10000	0.1	4.1	7.8	21	21
5000	0.2	7.8	14.9	44	39
2000	0.5	18.7	31.6	92	72
1000	1.0	32.1	43.3	139	91
500	2.0	41.8	49.3	173	97
200	5.0	47.7	51.7	188	108
50	20.0	49.5	52.3	211	154

Tab. 2: Measurement values for parallel circuit.

f / Hz	T / ms	U_{m0} / mV		Δt / μs	
		$N = 1200$ $L = 23.0$ mH	$N = 800$ $L = 10.4$ mH	$N = 1200$ $L = 23.0$ mH	$N = 800$ $L = 10.4$ mH
10000	0.1	60.0	61.7	1.3	2.8
5000	0.2	60.3	62.6	4.4	8.3
2000	0.5	63.0	73.0	25.0	45.0
1000	1.0	76.2	104.5	89.3	130.6
500	2.0	106.0	170.9	238.2	278.6
200	5.0	200.0	316.0	599.4	545.6
50	20.0	323.6	431.5	883.8	614.8

EVALUATION

- Calculate the sizes of the inductive impedance using the formula $X_{L0} = 2 \cdot \pi \cdot f \cdot L$ (see equation 1) and enter the results into Table 3.

- From the values of U_{m0} (tables 1, 2) and R_m (1 Ω), calculate the current amplitudes using the equation $I_0 = U_{m0} / R_m$ and enter the results into Table 3.

- Calculate the total impedance values Z_{0S} and Z_{0P} using the equation $Z_0 = U_0 / I_0$ ($U_0 = 6$ V) and enter the results into Table 3.

- From the values for the period of oscillation T and the time difference Δt (tables 1, 2) calculate the phase shifts using the equation $\varphi = 360° \cdot \Delta t / T$ and enter the results into Table 3.

- Plot the total impedance values Z_{0S} and Z_{0P} and phase shifts φ for the series and parallel circuits as a function of X_{L0} (Fig. 4 – 7).

- Calculate the theoretical total impedance values Z_{0S} and Z_{0P} and phase shifts φ according to equation (5) for the series circuit and equation (6) for the parallel circuit.

$$(9) \quad Z_{S0} = \sqrt{R^2 + X_{L0}^{\,2}} \;,\;\; \varphi = \arctan\left(\frac{X_{L0}}{R}\right)$$

$$(10)\; Z_{P0} = \frac{1}{\sqrt{\dfrac{1}{R^2} + \dfrac{1}{X_{L0}^{\,2}}}} \;,\;\; \varphi = \arctan\left(\frac{R}{X_{L0}}\right),$$

Draw them as continuous lines into Figs. 4 – 7.

Conclusion

If the inductive impedance is large, the resistance of the series circuit will have the value of the inductive resistance and the parallel circuit will have the value of the ohmic resistance. The phase shift is between 0° and 90° and equals 45° if the ohmic and inductive resistance values are the same.

Tab. 3: Calculation of absolute value X_{L0} of inductive impedance from measurements of current amplitude I_0, total impedance values Z_{0S} and Z_{0P} and phase shift φ for series and parallel circuits.

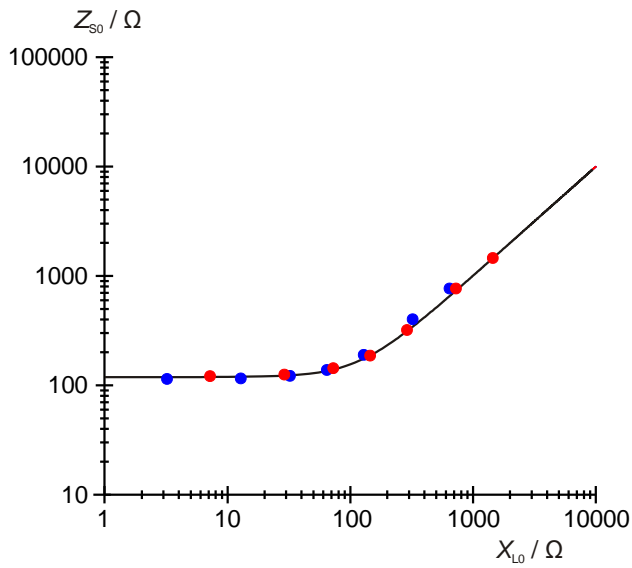| N L / mH	X_{L0} / Ω	Series circuit			Parallel circuit		
		I_0 / mA	Z_{S0} / Ω	φ	I_0 / mA	Z_{P0} / Ω	φ
1200 23.0	1445.1	4.1	1461.6	75.6°	60.0	100.1	4.7°
	722.6	7.8	766.5	79.2°	60.3	99.4	7.9°
	289.0	18.7	320.7	66.2°	63.0	95.2	18.0°
	144.5	32.1	187.1	50.0°	76.2	78.8	32.1°
	72.3	41.8	143.5	31.1°	106.0	56.6	42.9°
	28.9	47.7	125.7	13.5°	200.0	30.1	43.2°
	7.2	49.5	121.3	3.8°	323.6	18.5	15.9°
800 10.4	653.5	7.8	766.5	75.6°	61.7	97.3	10.0°
	326.7	15.0	402.9	70.2°	62.6	95.8	14.9°
	130.7	31.6	189.9	51.8°	72.9	82.3	32.4°
	65.3	43.3	138.4	32.8°	104.5	57.4	47.0°
	32.7	49.3	121.8	17.5°	170.9	35.1	50.1°
	13.1	51.7	116.0	7.8°	316.0	19.0	39.3°
	3.3	52.3	114.7	2.7°	431.5	14.0	11.1°

Fig. 4: Total impedance of series circuit for $N = 800$ (●) and $N = 1200$ (●) turns.
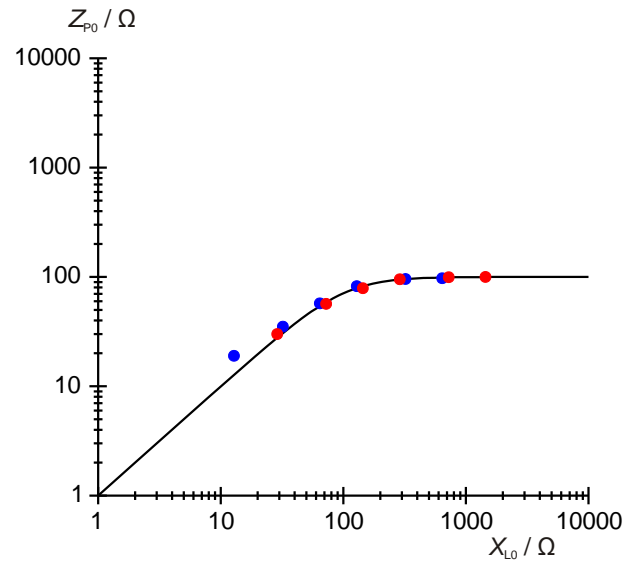

Fig. 6: Total impedance of parallel circuit for $N = 800$ (●) and $N = 1200$ (●) turns.
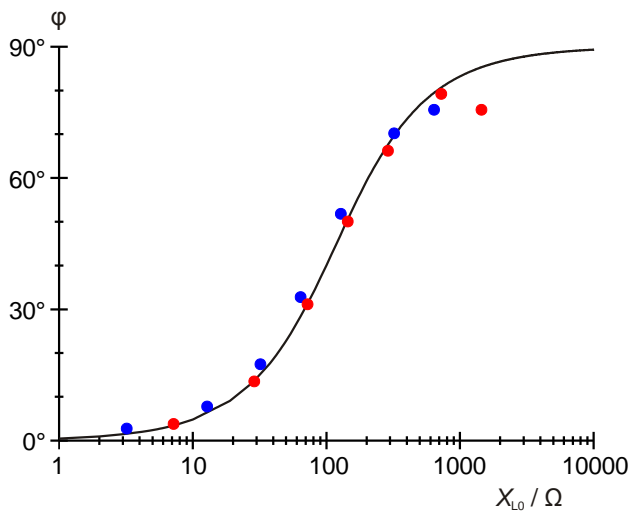

Fig. 5: Phase shift in series circuit for $N = 800$ (●) and $N = 1200$ (●) turns.
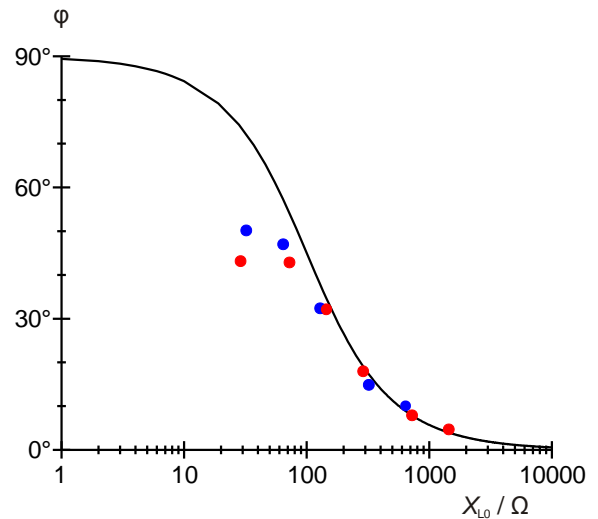

Fig. 7: Phase shift in series circuit for $N = 800$ (●) and $N = 1200$ (●) turns.