Optics

Wave Optics

3B
PHYSICS EXPERIMENT

Diffraction by Multiple Slits and Gratings

DEMONSTRATE THE WAVE NATURE OF LIGHT AND DETERMINE THE WAVELENGTH.

- Investigate diffraction at a pair of slits with different distances between the slits.

- Investigate diffraction at a pair of slits with different slit widths.

- Investigate diffraction by multiple slit systems with different numbers of slits.

- Investigate diffraction by a line grating and a lattice grating.

UE4030200

11/15 JS/UD

Fig. 1: Measurement set-up.

BASIC PRINCIPLES

The diffraction of light by multiple slits or a grating can be described by considering the superimposition of individual components of the coherent wave radiation, which emerge from each point of illumination formed by the multiple slits, according to the Huygens principle. The superimposition leads to constructive or destructive interference in particular directions, and this explains the pattern of bright and dark bands that is observed beyond the system of slits.

In the space beyond a pair of slits, the light intensity at a particular angle of observation α_n is greatest when, for each individual wave component coming from the first slit, there exists an exactly similar wave component from the second slit, and the two interfere constructively. This condition is fulfilled when the path difference Δs_n between two wave components emerging from the centres of the two slits is an integral multiple of the wavelength λ of the light (see Fig. 2), thus:

$$(1) \quad \Delta s_n(\alpha_n) = n \cdot \lambda$$

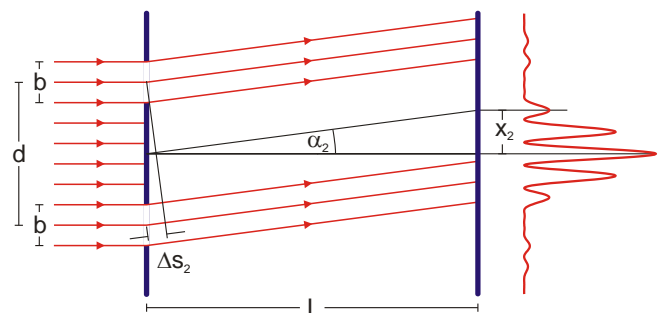where $n = 0, \pm 1, \pm 2, \ldots$ is called the diffraction order.

Fig. 2: Schematic diagram of the diffraction of light at a pair of slits (b: slit width, d: distance between slits, L: distance from slits to observation screen, x_2: distance of second maximum from centre, α_2: diffraction angle for the second maximum, Δs_2: path difference for rays at the midpoint).

At large distances L from the pair of slits and for small angles of observation α_n, the relationship between the path difference Δs_n and the position coordinate x_n of the nth-order intensity maximum is:

$$(2) \quad \frac{\Delta s_n}{d} = \sin\alpha_n \approx \tan\alpha_n = \frac{x_n}{L}$$

d: distance between the slits.

Thus the maxima are spaced at regular intervals with a separation a given by:

$$(3) \quad a = x_{n+1} - x_n = \frac{\lambda}{d} \cdot L.$$

This relationship is also valid for diffraction at a multiple slit system consisting of N equidistant slits ($N > 2$). Equation (1) states the condition for constructive interference of the wave elements from all N slits. Therefore, Equations (2) and (3) can also be applied to a multiple slit system.

The mathematical derivation of the positions of the intensity minima is more difficult. Whereas in the case of a pair of slits there is an intensity minimum exactly halfway between two intensity maxima, for the multiple slits system a minimum is observed between the nth and the (n+1)th maxima when the wave components from the N slits interfere in such a way that the total intensity is zero. This occurs when the path difference between the wave components from the centres of the slits satisfies the condition:

$$(4) \quad \Delta s = n \cdot \lambda + m\frac{\lambda}{N}$$

where $n = 0, \pm 1, \pm 2, \ldots,\ m = 1, \ldots, N-1$

Therefore N -1 minima are visible and between them are N - 2 "minor maxima" with intensities smaller than those of the principal maxima.

As the number of slits N is progressively increased, the contribution of the minor maxima gradually disappears. Then the system is no longer described as a multiple slit system but as a line grating. Finally, a lattice grating can be regarded as an arrangement of two line gratings, one rotated at 90° relative to the other. The diffraction maxima now become points on a rectangular grid with a spacing interval given by Equation (3).

The intensity (brightness) of the principal maxima is modulated according to the intensity distribution function for diffraction at a single slit. The greater the slit width b, the greater the concentration of intensity towards smaller values of the angle α. For an exact derivation it is necessary to sum the amplitudes of all the wave components, taking into account the path differences, to obtain the total amplitude A. At a point on the screen defined by x, the intensity is:

$$(5) \quad I = A^2 \propto \left(\frac{\sin\left(\frac{\pi \cdot b}{\lambda} \cdot \frac{x}{L}\right)}{\frac{\pi \cdot b}{\lambda} \cdot \frac{x}{L}} \right)^2 \cdot \left(\frac{\sin\left(N \cdot \frac{\pi \cdot d}{\lambda} \cdot \frac{x}{L}\right)}{\sin\left(\frac{\pi \cdot d}{\lambda} \cdot \frac{x}{L}\right)} \right)^2 = f(x).$$

Function $f(x)$ on the right-hand side of equation (5) is given by the following limit expression at position $x = 0$ at the centre of the diffraction pattern:

$$(6) \quad \lim_{x \to 0} f(x) = N^2$$

The first factor of $f(x)$ describes the diffraction by a single slit and the second takes into account interference between a number of slits N.

LIST OF EQUIPMENT

1	Laser Diode, Red	U22000	1003201
1	Optical Bench K, 1000 mm	U8475240	1009696
2	Optical Rider K	U8475350	1000862
1	Clamp K	U84755401	1008518
1	Holder K for Diode Laser	U8475550	1000868
1	Diaphragm with 3 Double Slits of Different Widths	U14100	1000596
1	Diaphragm with 4 Double Slits of Different Spacings	U14101	1000597
1	Diaphragm with 4 Multiple Slits and Gratings	U14102	1000598
1	Diaphragm with 3 Ruled Gratings	U14103	1000599
1	Diaphragm with 2 Cross Gratings	U14106	1000601

SET-UP AND PROCEDURE

- Set up one optical rider on the optical bench at 0 cm and one at 70 cm and secure them in place (Fig. 1).

- Insert the laser diode into its holder and secure it in place. Insert the holder with the laser inside it into the optical rider at 70 cm.

- Insert the clamp into the optical rider at the 0-cm position. Fix the diffraction diaphragms into the clamp in such a way the diffracting object is centred on the optical.

- Position a projection screen at a distance $L \approx 7$ m from the diffracting object (Fig. 2) at right angles to the optical axis. Measure the precise distance between the diffracting object and the projection screen and make a note of it. Do not change the position of the optical bench subsequent to this. If necessary, though, the distance L may be re-measured each time the diaphragm/diffraction object is replaced.

- Connect the plug-in power supply of the laser diode to the mains and turn on the laser.

- Clamp all the diaphragms with diffracting objects into the clamp on the optical bench one after another and view the brightness distribution for each of them on the projection screen, taking photographs of the results if required.

- For each of the 4 double slits with differing slit separations, measure the distance x_n between a maximum of order $n \neq 0$ and the 0-order maximum in the centre of the diffraction pattern (an example for $n = 2$ can be seen in Fig. 2) and enter the result into Table 1 (see evaluation section).
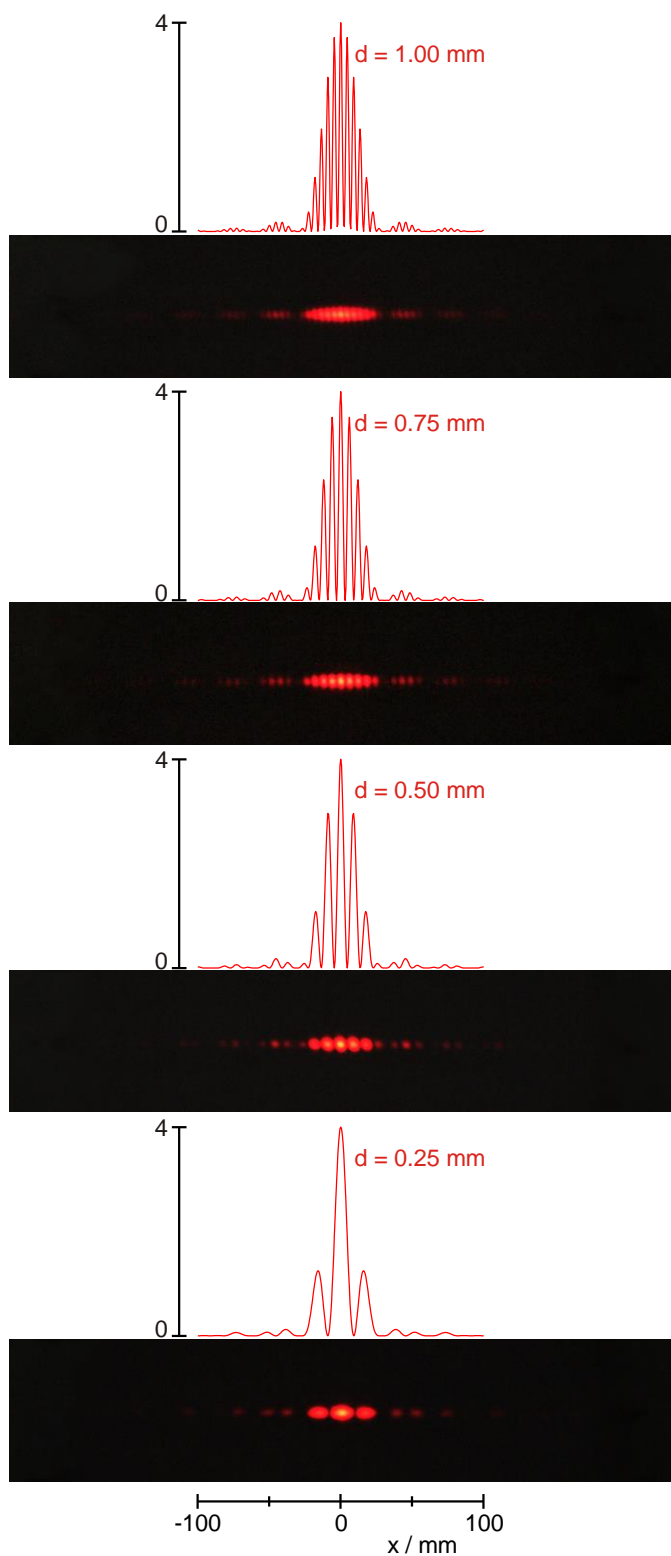
SAMPLE MEASUREMENT

Fig. 3: Diffraction from pairs of slits with various slit separations d. Calculated function $f(x)$ and observed intensity. Number of slits $N = 2$, width of slits $b = 0.15$ mm.
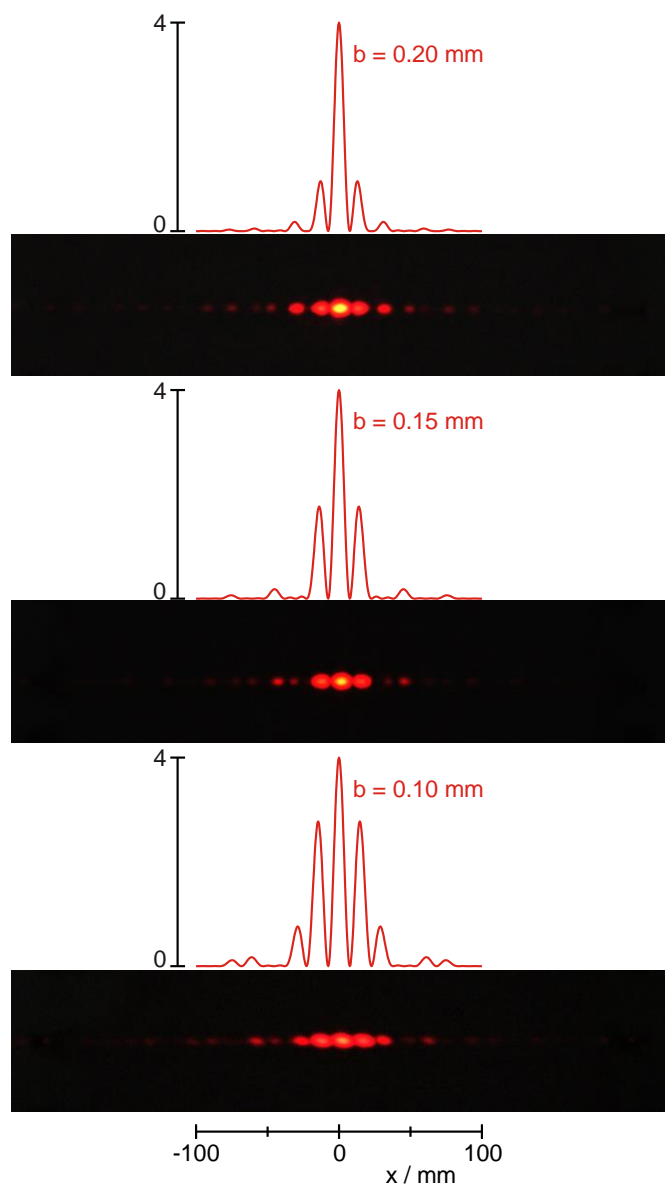
Fig. 4: Diffraction from pairs of slits with various slit widths b. Calculated function $f(x)$ and observed intensity. Number of slits $N = 2$, separation of slits $d = 0.30$ mm.
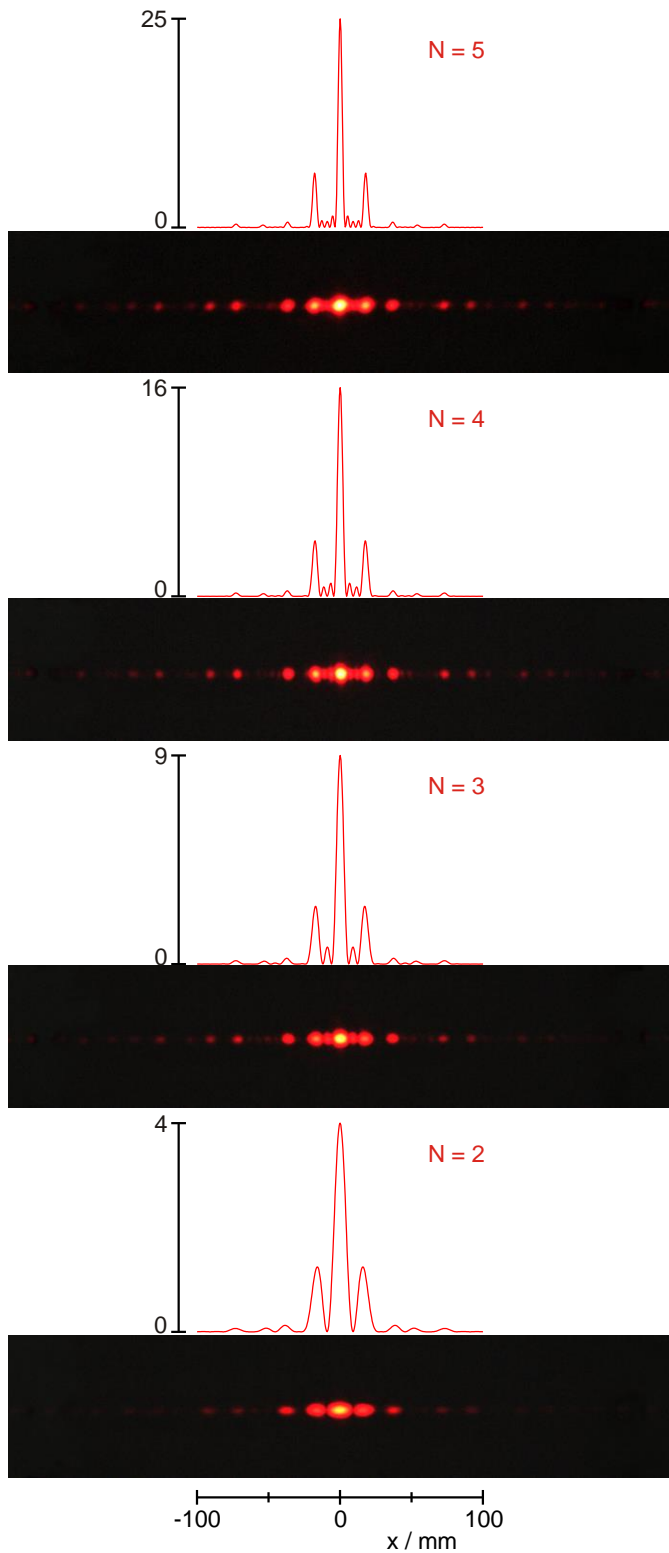
Fig. 5: Diffraction from multiple slits with various numbers of slits *N*. Calculated function *f*(*x*) and observed intensity. Slit separation *d* = 0.25 mm, width of slits *b* = 0.15 mm.
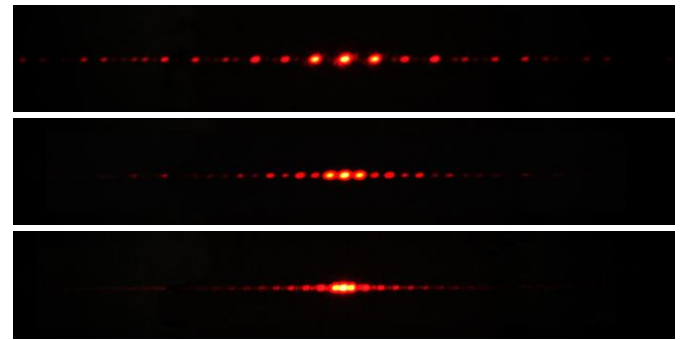

Fig. 6: Diffraction from ruled gratings with 20 (bottom), 40 (centre) and 80 lines/cm (top), corresponding grating constants *g* = 0.50, 0.25 and 0.125 mm.
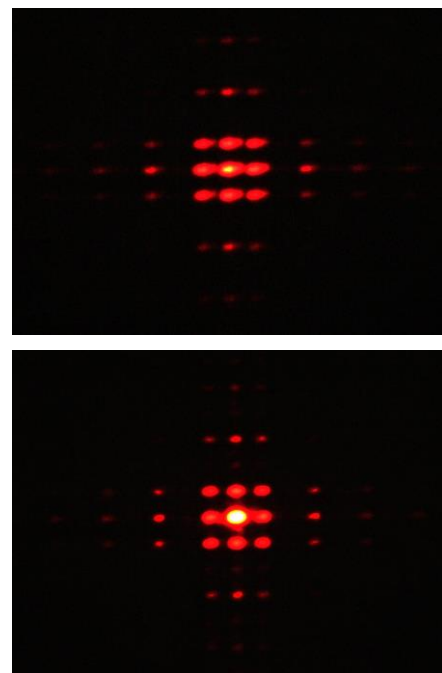

Fig. 7: Diffraction from white (bottom) and black (top) cross-ruled gratings with 40 lines/cm, corresponding to grating constant *g* = 0.25 mm.

EVALUATION

In order to evaluate the diffraction patterns observed on the screen resulting from the various slit configurations, the distributions of brightness and the function which describes them $f(x)$ are derived in accordance with equation (5) using various pre-defined widths, separations and numbers of slits and compared with what actually appears on the screen (Figs. 3, 4 and 5).

Where slits of varying separation are used (Fig. 3), it can be seen that the number of interference maxima increases as the separation becomes greater and the width of the slits becomes narrower, since the width of the diffraction maxima (the envelopes encompassing the maxima) remains the same. With increasing separation, it is possible for more diffracted waves to interfere with one another.

Where slits of varying width are used (Fig. 4) it can be seen that the number of interference maxima stays the same as the separation becomes greater but brightness of lines with order $n \neq 0$ decreases in intensity because the widths of the diffraction maxima (the envelopes encompassing the maxima) become narrower. Due to the less prominent diffraction arising from wider slits, the interference between diffracted waves is less prominent as well.

With the multiple slit (Fig. 5), as expected, N-2 "minor" maxima can be observed, i.e. a double slit does not result in any, a triple slit results in one and five slits result in three.

In general, at places where there are diffraction minima, no interference maxima can be observed. This is the case if the first term of $f(x)$ in equation (5) is zero, i.e. for integer multiples of $x = (\lambda / b) \cdot L$. For $b = 0.15$ mm, for example, the following is the case (see Fig. 3 and Fig. 5):

$$(7) \quad x = \frac{\lambda}{b} \cdot L = \frac{650\,\text{nm}}{0,15\,\text{mm}} \cdot 7\,\text{m} = 30,3\,\text{mm}$$

For gratings with parallel ruled lines (Fig. 6), the effect of the "minor maxima" disappears, as expected, and the separation a between principal maxima on the screen becomes greater, as per equation (3), with increasing numbers of lines/cm and the corresponding decreasing grating constant g.

For the two cross-ruled gratings (Fig. 7) the diffraction maxima become, again as expected, points on a rectangular grid. The diffraction maxima from the white grating appear to be brighter than from the black one because the white grating allows more light to pass through it and therefore absorbs less of the intensity than the black one does.

The wavelength of the diffracted light can be determined for double slits of varying slit separation from the regular separation a between maxima by means of equation (3).

- Work out the quotients L/d for the 4 double slits with various slit separations (Table 1).

- To determine the separation a between maxima, divide the measured separations x_n by the order of diffraction n in each case (Table 1).

- Plot the values of a determined by measurement in a graph against the quotients L/d and draw a line which fits the points (Fig. 8).

According to equation (3), the gradient of the best-fitting straight line should be exactly equal to the wavelength λ:

$$(8) \quad a = \lambda \cdot \left(\frac{L}{d}\right)$$

A value of $\lambda = 630$ nm results for the wavelength, which is within 3% of the specified value $\lambda = 650$ nm.

Tab. 1: Diffraction from double slits with different slit separations d, cf. Fig. 3 ($L = 7$ m: distance to screen, n: order of diffraction, x_n: measured separation, a: separation of maxima determined from x_n).

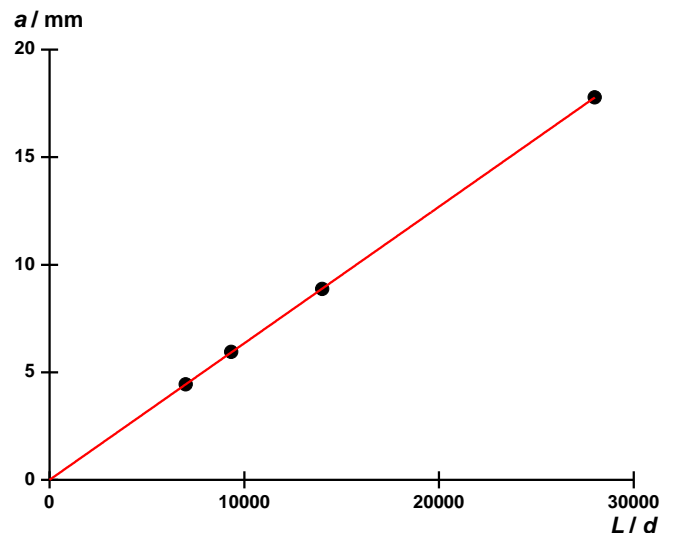| d / mm	L / d	n	x_n / mm	a / mm
0,25	28000	2	35,56	17,78
0,50	14000	6	53,20	8,87
0,75	9333	8	47,60	5,95
1,00	7000	12	53,20	4,43

Fig. 8: Separation between maxima a as a function of quotients L/d of the distance to the screen $L = 7$ m and the separation of slits d.